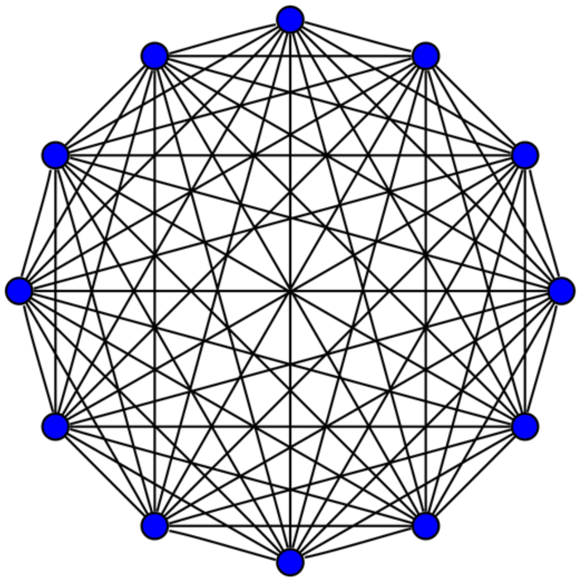
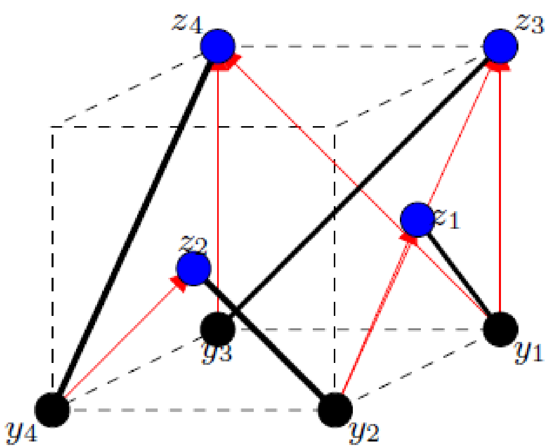
Spacetime Replication of Continuous Variable Quantum Information





Grant Salton
Stanford University

With: Patrick Hayden, Sepehr Nezami, and Barry Sanders

arXiv: 1501.####

Outline

- Part 1: Replicating information in spacetime
  - Complete characterization of which spacetime regions can contain the same quantum information
  - Quantum error correcting code to realize any allowed configuration of regions
- Part 2: Continuous variable codes
  - A general CV code for any allowed configuration
  - A specific code for a simple, yet non-trivial configuration (+ an optical implementation!)

Quantum Information Bedrock

Information cannot propagate faster than light – *no signaling*

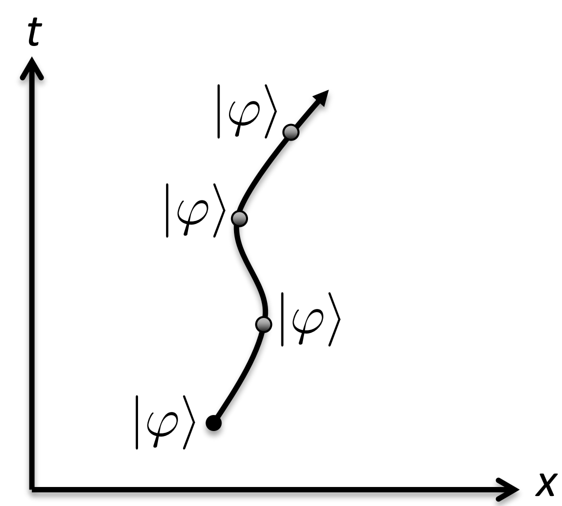
Quantum information *cannot be cloned*.

$$|\varphi\rangle \not\to |\varphi\rangle|\varphi\rangle$$

Quantum information cannot be replicated on a *spatial slice*.

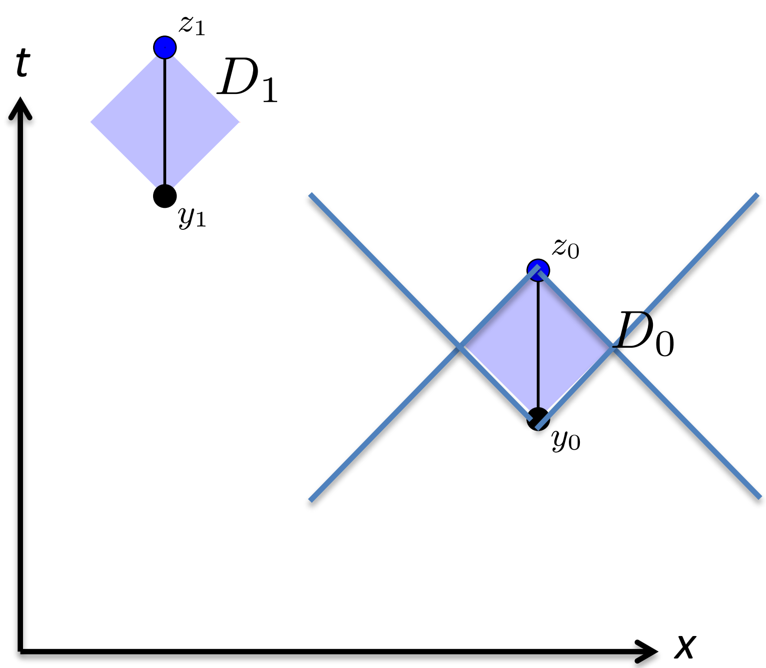
Quantum information **must** be widely replicated in space*time*.

And yet…



Hayden and May precisely characterized which forms of replication are possible.

Replicating info in causal diamonds

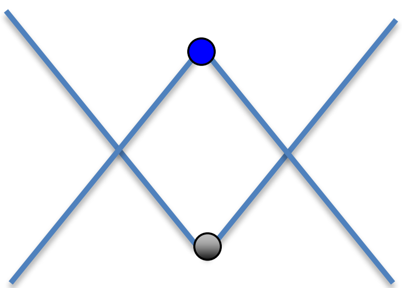


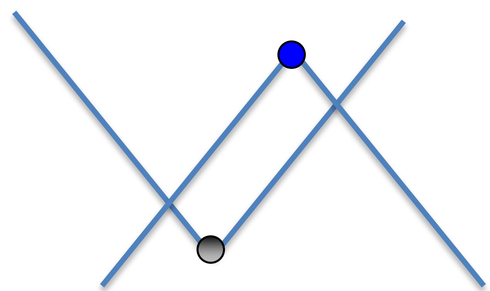
Define *causal diamond* D_j to be the intersection of the future of y_j and the past of z_j.

D_j consists of the points that can both be affected by an event at y_j and can affect the state at z_j.

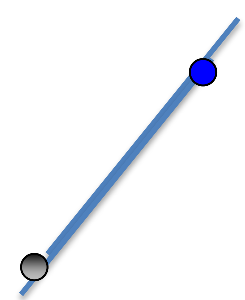
Hayden and May: Replication is possible iff every pair of causal diamonds is *causally related*: *i.e.*, there exists a causal curve from D_i to D_j or vice-versa.

Causal diamond geometry

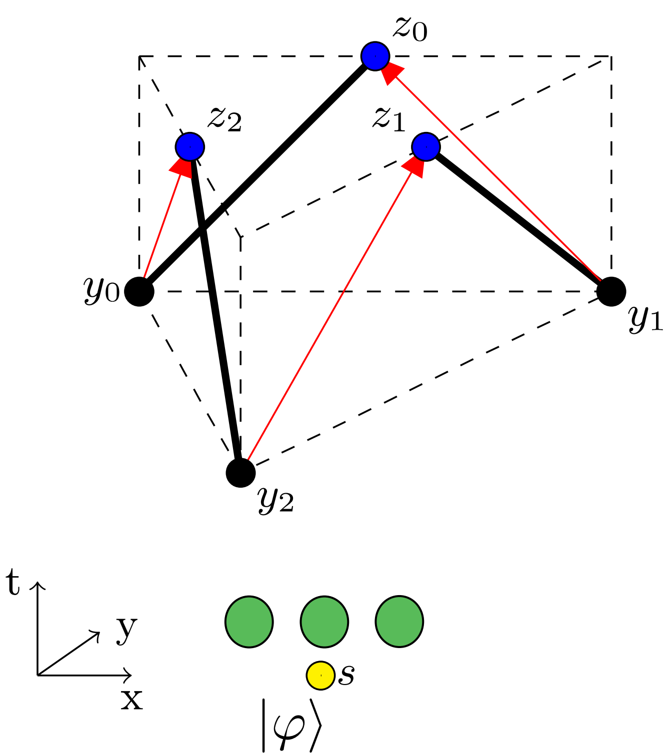




Diamond becomes a line segment when top and bottom are lightlike separated:



The causal merry-go-round



φ is encoded into ((2,3)) threshold quantum error correcting code at s

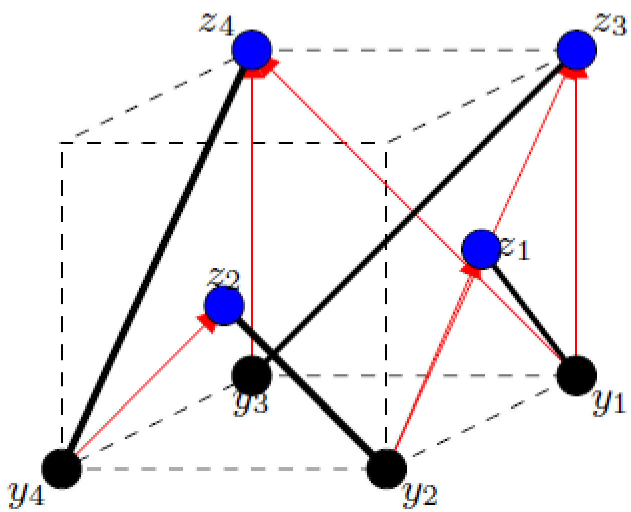
One share sent to each of y_j

Each share is then sent at the speed of light along a red ray

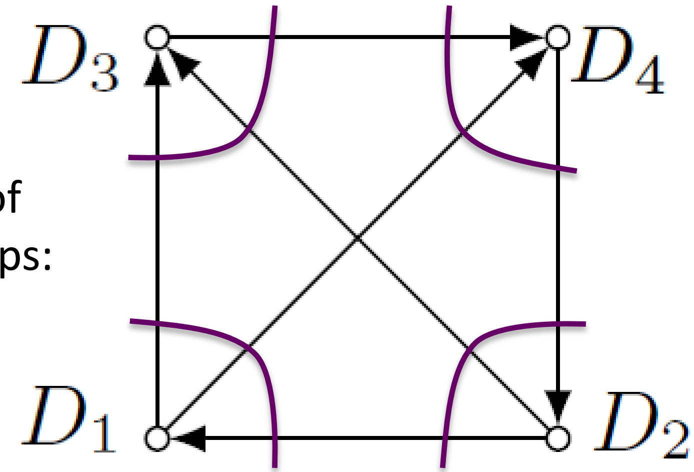
2 share pass through each causal diamond $y_j z_j$

The same quantum information is replicated in each causal diamond

General procedure



$G = (V,E)$ graph of causal relationships:



Encode φ into a quantum error correcting code with one share for each edge.

Transport each edge share according to directed edge in the graph

Code property: φ can be recovered provided all the shares associated to any D_j

Then all shares required to recover φ at D_j pass through D_j .

Unusual QEC: ~n^2 qubits but recovery using (n-1). Vanishing fraction $O(1/n)$.

Outline

- Part 1: Replicating information in spacetime
  - Complete characterization of which spacetime regions can contain the same quantum information
  - Quantum error correcting code to realize any allowed configuration of regions
- Part 2: Continuous variable codes
  - A general CV code for any allowed configuration
  - A specific code for a simple, yet non-trivial configuration (+ an optical implementation!)

Continuous variable quantum information

Continuous variables are a promising, experimentally feasible avenue for explicit demonstration of information replication

Existing experimental progress along the lines of our proposed scheme

The CV code we propose is more efficient than the qubit code just described

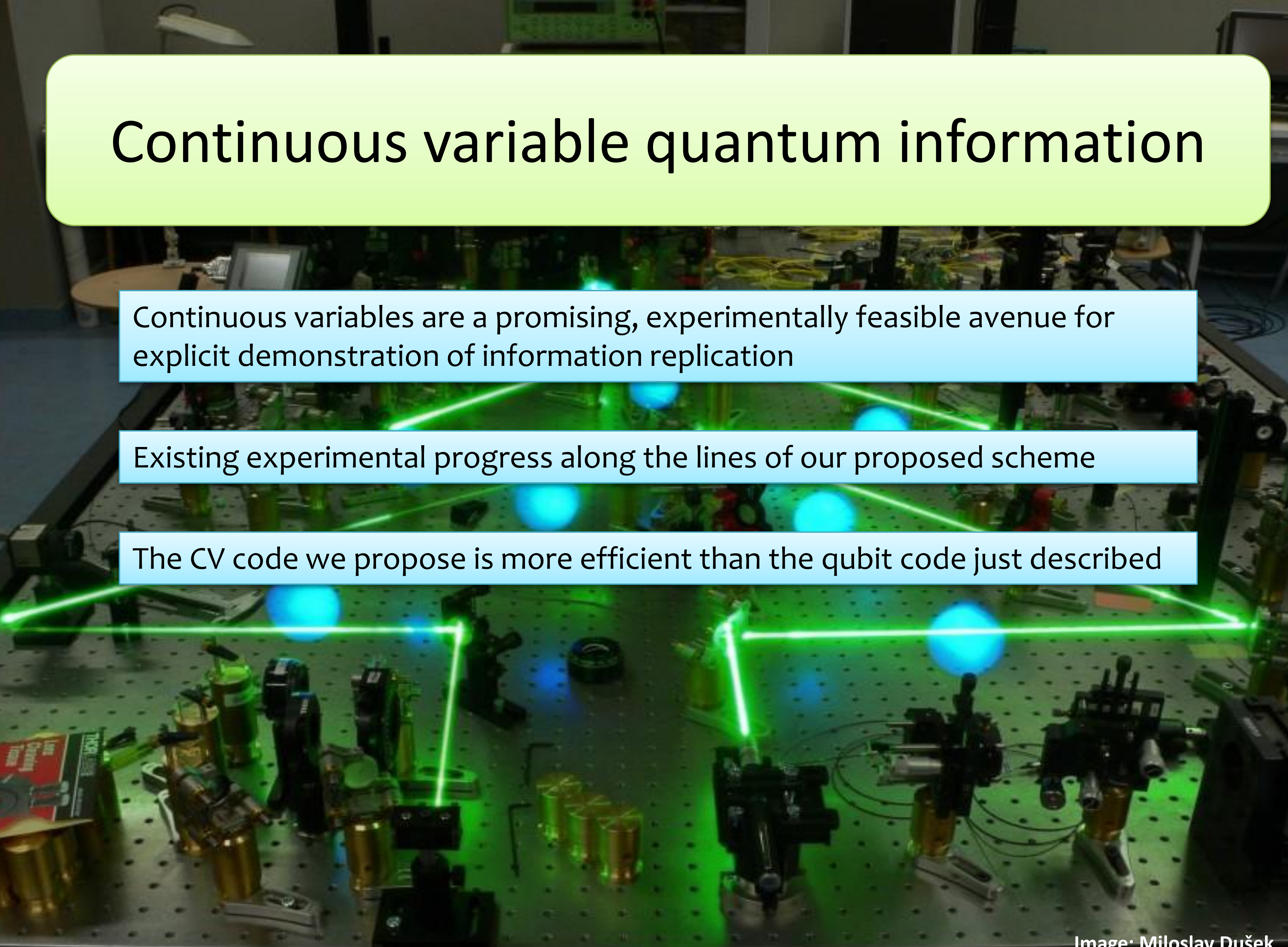


Image: Miloslav Dušek

Continuous Variable Quantum Info

Encode our state in a continuous variable degree of freedom: *optical mode.*

Generalize the Pauli group ----- Heisenberg-Weyl group

$$U(\boldsymbol{s},\boldsymbol{t}) = \exp(i(\boldsymbol{s}\cdot\boldsymbol{X} + \boldsymbol{t}\cdot\boldsymbol{P})) \qquad \mathbf{s},\mathbf{t} \in \mathbb{R}^n$$

n bosonic modes. Each mode has two quadratures

$\boldsymbol{X} = (X_1, \ldots, X_n)^\top$ Generators of translations (displacements) in *p* quadrature

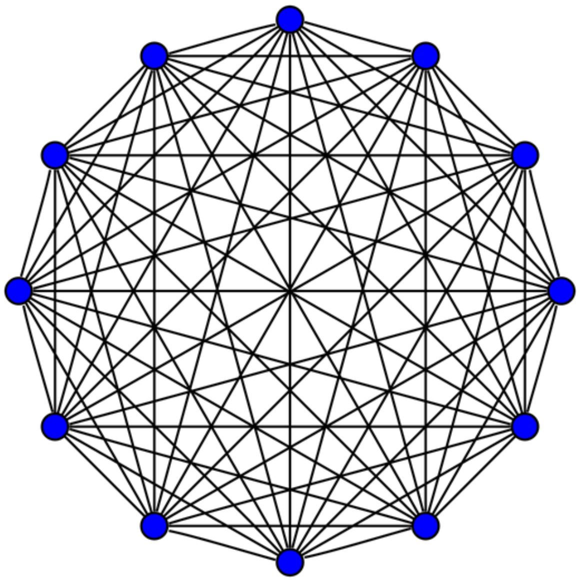
$\boldsymbol{P} = (P_1, \ldots, P_n)^\top$ Generators of translations (displacements) in *x* quadrature

CV Code for general replication

Suppose we have N spacetime regions in which we want to perform information replication:

Construct the graph of causal relations (complete graph, N vertices)

Assign one mode per edge $n = \binom{N}{2}$ modes

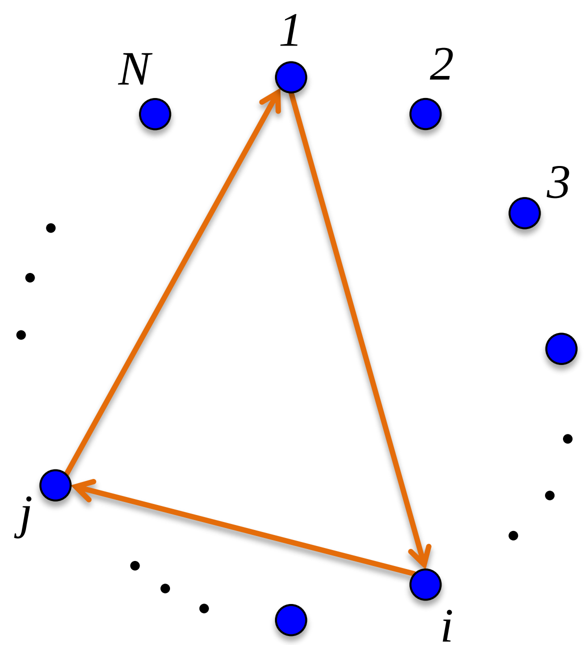


The code is actually a CSS code, so we build the X-type and P-type generators separately

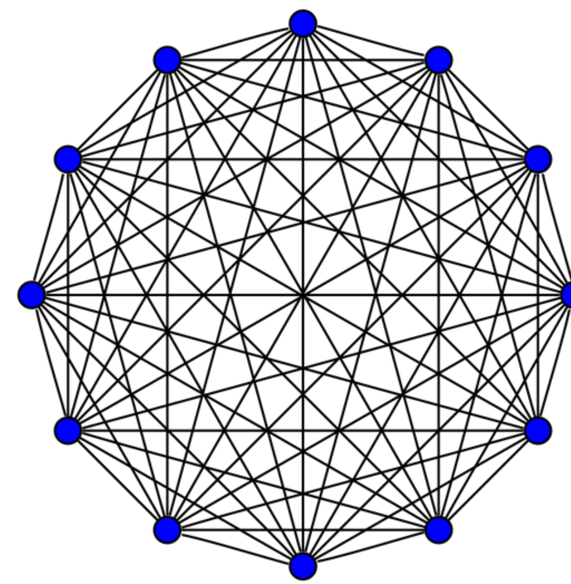
Motivated by homology

CV Code for general replication

X-type stabilizer generators are triangular subgraphs including vertex 1:

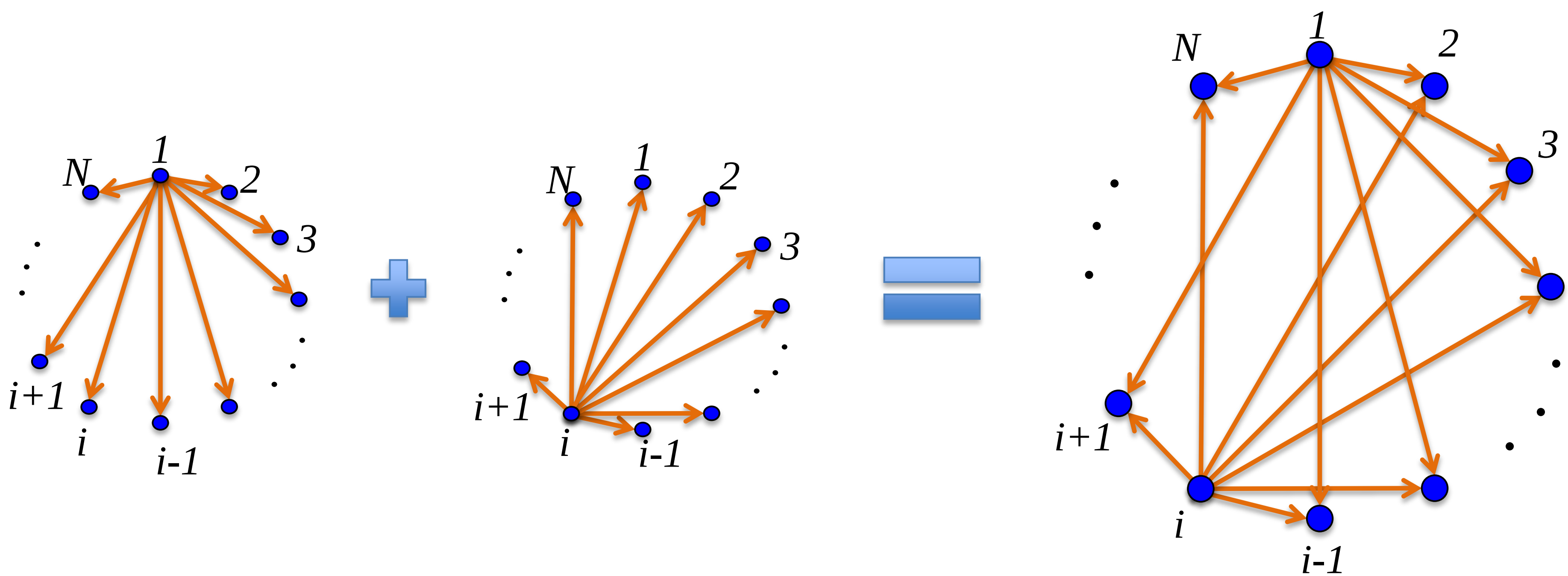


$$g_{ij} = \exp(i\boldsymbol{v_{ij}} \cdot \boldsymbol{X}), \quad (2 \le i < j \le N).$$



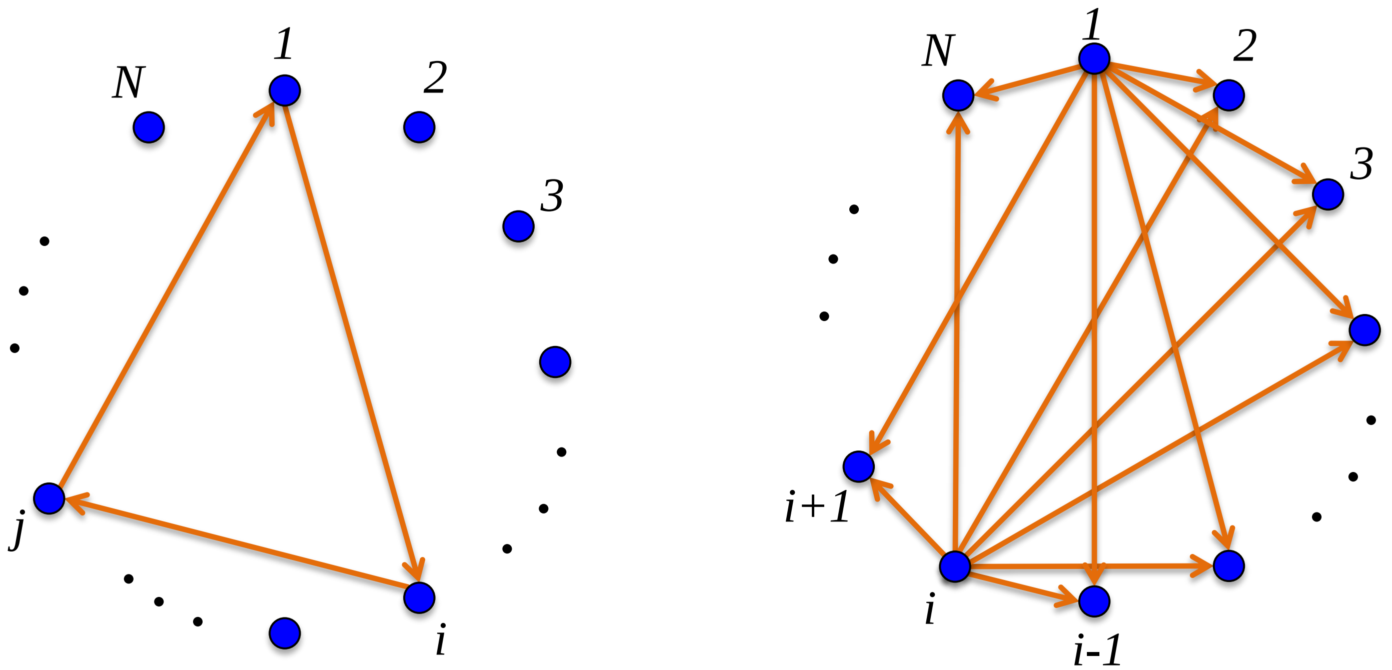
CV Code for general replication

P-type stabilizer generators are also subgraphs:



$$h_i = \exp(i\boldsymbol{w_i} \cdot \boldsymbol{P}), \quad (2 \leq i \leq N-1)$$

CV Code for general replication

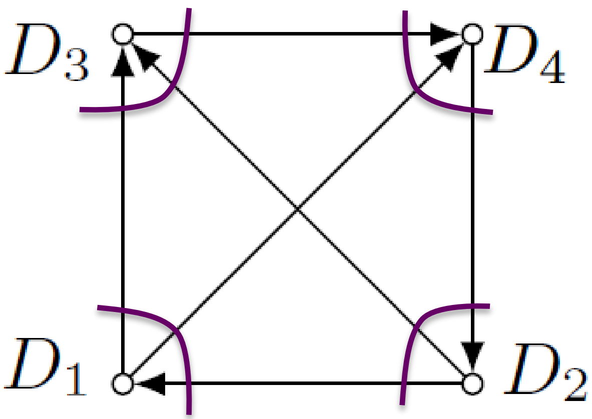


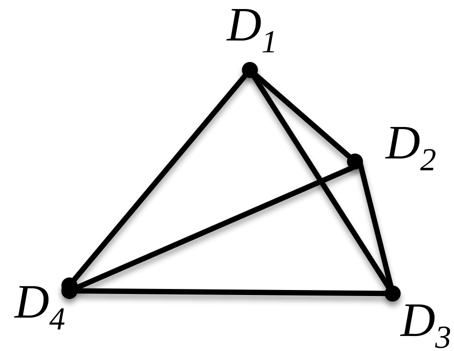
$$g_{ij} = \exp(i\boldsymbol{v_{ij}} \cdot \boldsymbol{X}), \quad (2 \le i < j \le N). \qquad h_i = \exp(i\boldsymbol{w_i} \cdot \boldsymbol{P}), \quad (2 \le i \le N-1)$$

$$S = \left\langle g_{23}^{a_1}, g_{24}^{a_2}, \ldots, g_{N-1,N}^{a_{\binom{N-1}{2}}}, h_2^{b_1}, h_3^{b_2}, \ldots, h_{N-1}^{b_{N-2}} \right\rangle$$

for all arbitrary $a_k, b_k \in \mathbb{R}$.

Motivation for the general code





$$C_4 \xrightarrow{\partial} C_3 \xrightarrow{\partial} C_2 \xrightarrow{\partial} C_1 \xrightarrow{\partial} C_0$$

$$U(\boldsymbol{s},\boldsymbol{t}) = \exp(i(\boldsymbol{s}\cdot\boldsymbol{X} + \boldsymbol{t}\cdot\boldsymbol{P})) \qquad \mathbf{s},\mathbf{t} \in \mathbb{R}^n$$

Code is subspace of wavefunctions stabilized by subgroup of commuting operators:

$$g_{ij} = \exp(i\boldsymbol{v_{ij}}\cdot\boldsymbol{X}) \qquad h_i = \exp(i\boldsymbol{w_i}\cdot\boldsymbol{P}),$$

Choose:

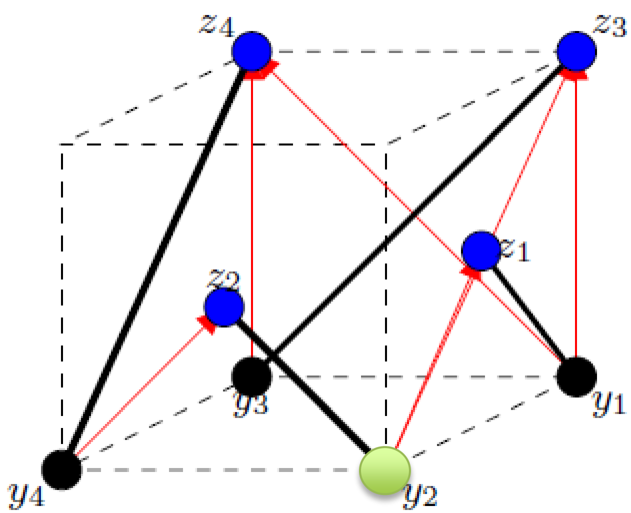
$v_{ij} = \partial f_{1ij} \quad \text{for } f_{1ij} \in C_3$

$w_k = \partial^T(\omega_1 + \omega_k) \quad \text{for } \omega_k \in C_1$

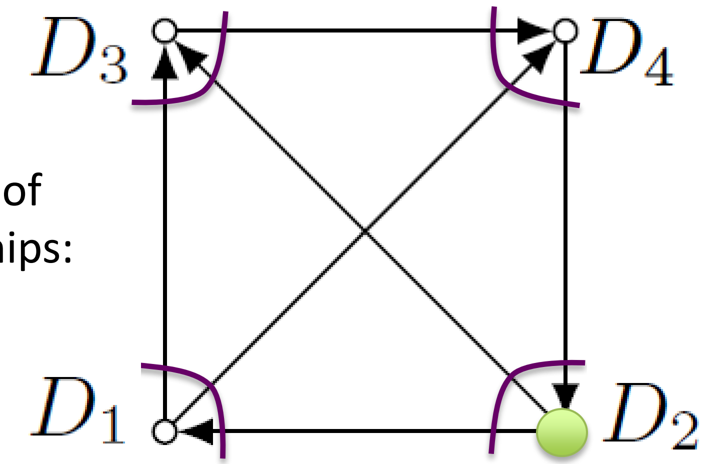
Commutativity condition:

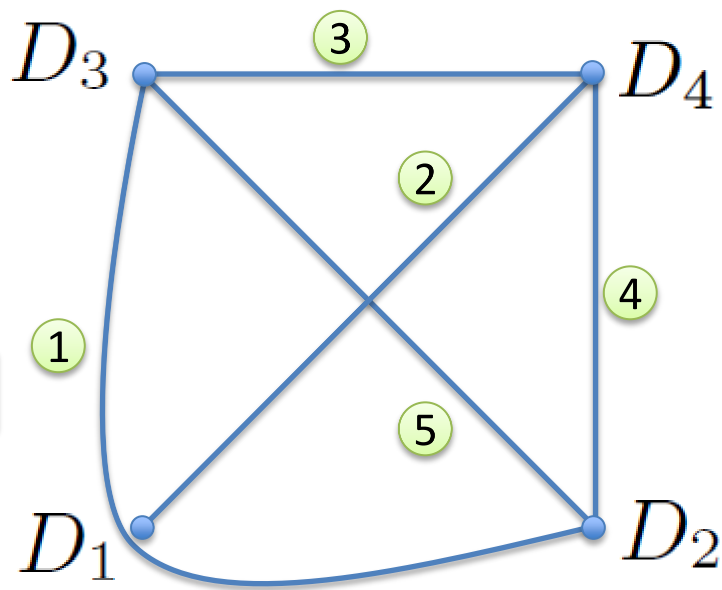
$$\begin{aligned} v_{ij}\cdot w_k &= (\partial f_{1ij})\cdot\partial^T(\omega_1+\omega_k) \\ &= (\partial^2 f_{1ij})\cdot(\omega_1+\omega_k) \\ &= 0 \end{aligned}$$

Four regions



G = (V,E) graph of causal relationships:





Our general code uses **six** modes to complete the task

BUT! We can bring this number down using a property of the physical configuration

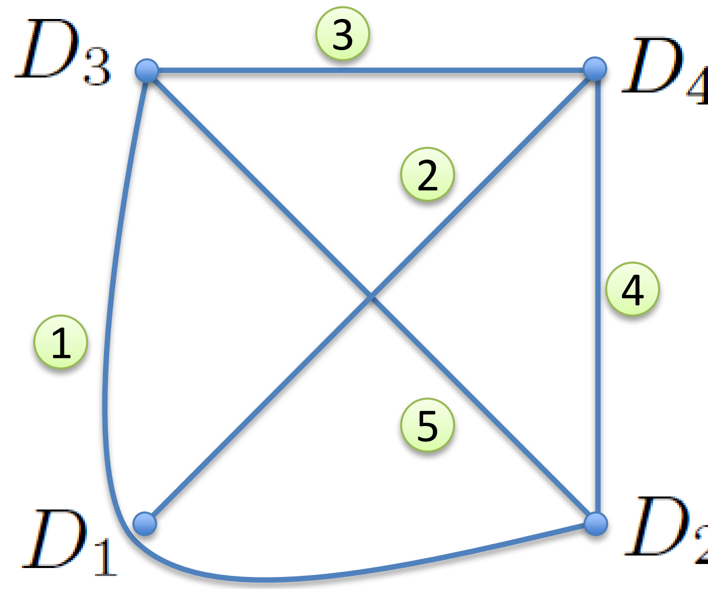
Use the ability of one share to traverse three diamonds

A five mode code

X P

$$
\begin{array}{ccccc|ccccc}
1 & 2 & 3 & 4 & 5 & 1 & 2 & 3 & 4 & 5 \\
\hline
\end{array}
$$

$$
\left(\begin{array}{ccccc|ccccc}
-1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & -2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1
\end{array}\right)
$$



$$|\alpha\rangle_{\text{enc}} = \int dx dy \, |x+\alpha, x-\alpha, x-y, x+y, y\rangle$$

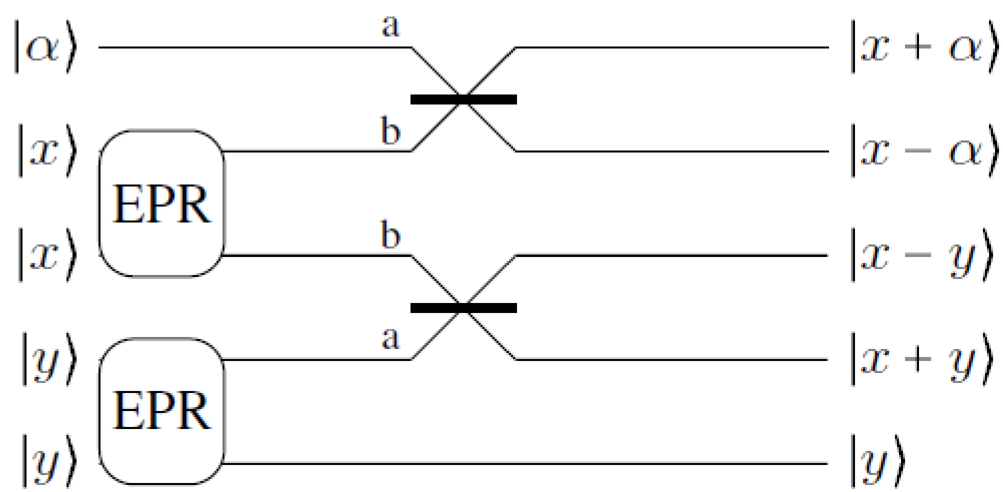
x eigenstate

The error model:
Loss of a known subset of modes

Equivalent to arbitrary displacements on the ‘lost’ modes

Optical implementation

Encoding

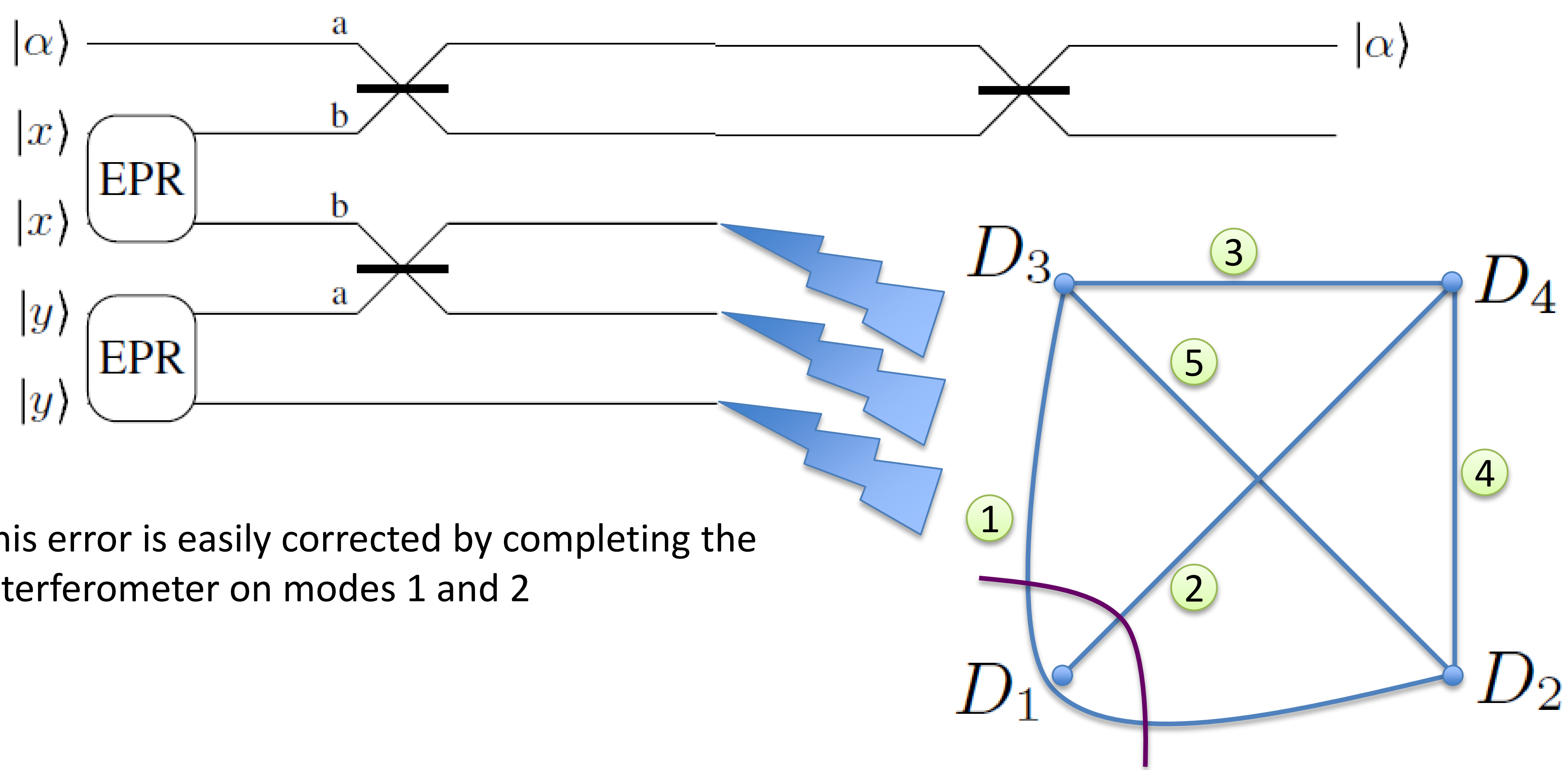


Encoding requires two sets of entangled photons and passive beamsplitters

We then carry out the replication task and need to recover the state $|\alpha\rangle$ using a known subset of the modes.

Optical implementation

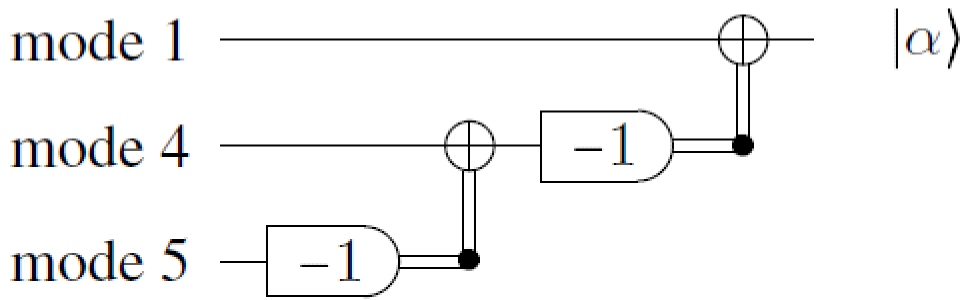
Decoding using only modes 1 and 2

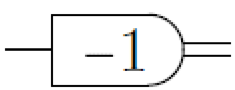


This error is easily corrected by completing the interferometer on modes 1 and 2

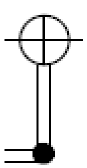
Optical implementation

Recovery from loss of modes 2 and 3



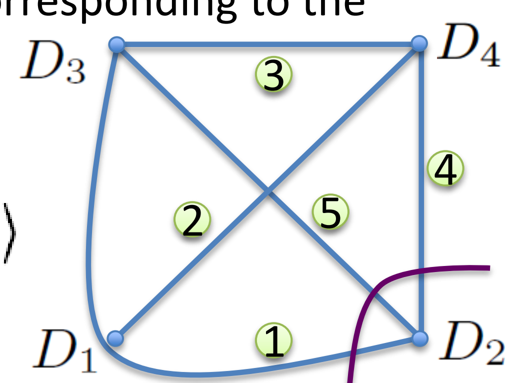


Is a measurement of the quadrature of the mode, followed by a rescaling of the resulting classical data by a factor of -1



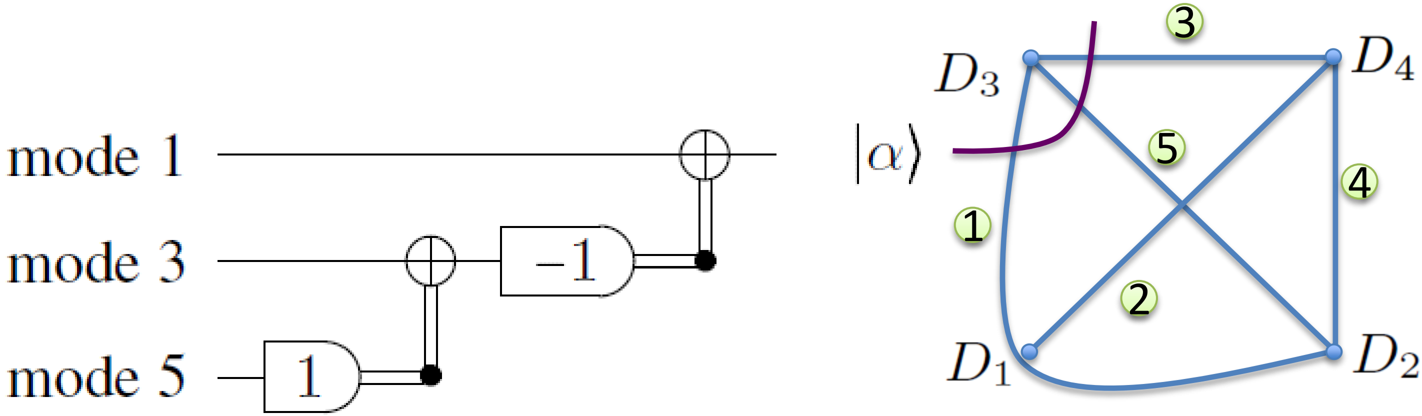
Is a displacement of the upper mode by an amount corresponding to the classical data in the lower mode

$$|\alpha\rangle_{\text{enc}} = \int dx dy\, |x+\alpha, x-\alpha, x-y, x+y, y\rangle$$

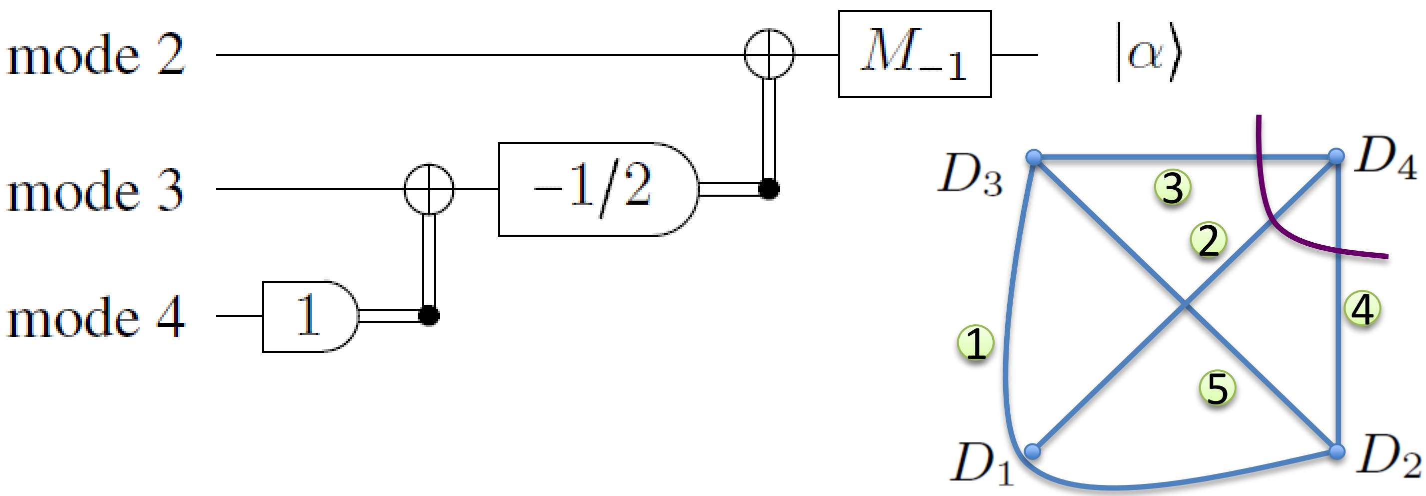


Optical implementation

Recovery from loss of modes 2 and 4



Recovery from loss of modes 1 and 5



Conclusions: Part 2

- Continuous variable code
  - More efficient than qubit code
  - Based on ideas from homology

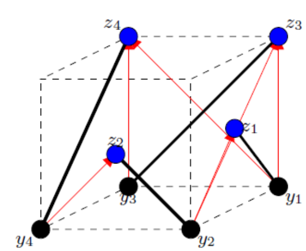
- Specific 5 mode code
  - *ad hoc* construction
  - Optical implementation
    - Design optical apparatus capable of demonstrating spacetime information replication

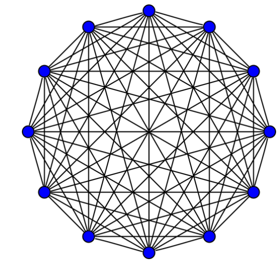
Next steps…

- Characterize the codes that arise when we have redundancy in the graph of causal structure

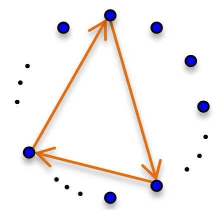
- Recruit experimentalists!

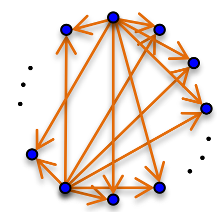
Summary

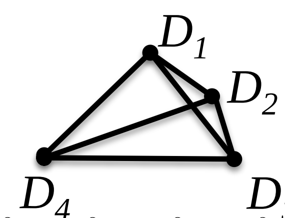




- Information replication
  - Complete characterization of the allowed configurations
  - Only constrained by no-cloning & no-signaling
  - Realized with QEC!







$C_4 \xrightarrow{\partial} C_3 \xrightarrow{\partial} C_2 \xrightarrow{\partial} C_1 \xrightarrow{\partial} C_0$

- Continuous variables
  - General solution in terms of CV code based on homology
  - Specific 5 mode code complete with optical implementation