Thermalization time bounds for Pauli stabilizer Hamiltonians

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California Institute of Technology

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$$t_{mix} \leq \mathcal{O}(N^2 e^{2\beta\overline{\epsilon}})$$

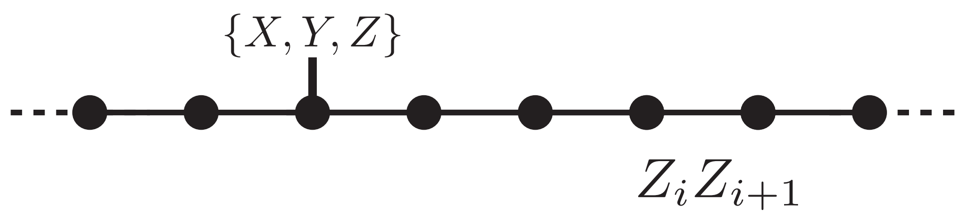
Overview

- Motivation & previous results
- Mixing and thermalization
- The spectral gap bound
- Proof sketch

Thermalization in Kitaev's 2D model

- Spectral gap bound for the 2D toric code and 1D Ising

R. Alicki, M. Fannes, M. Horodecki J. Phys. A: Math. Theor. 42 (2009) 065303



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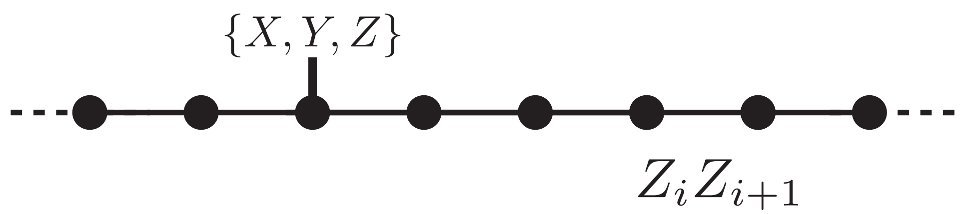

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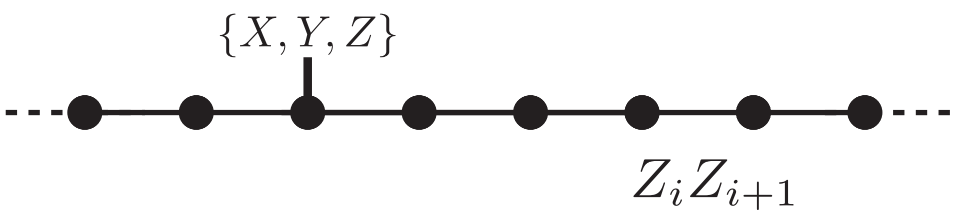

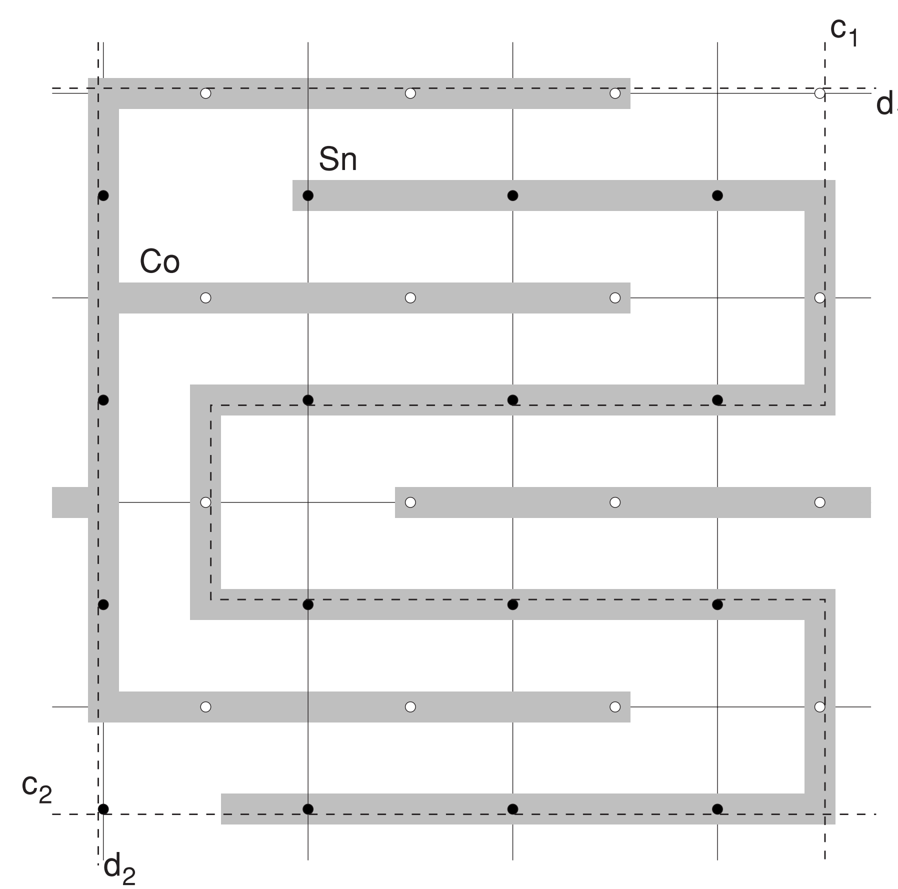

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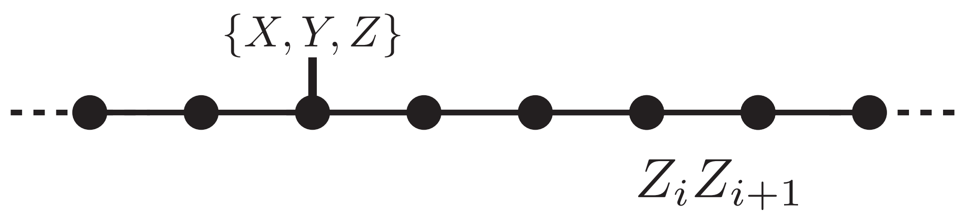

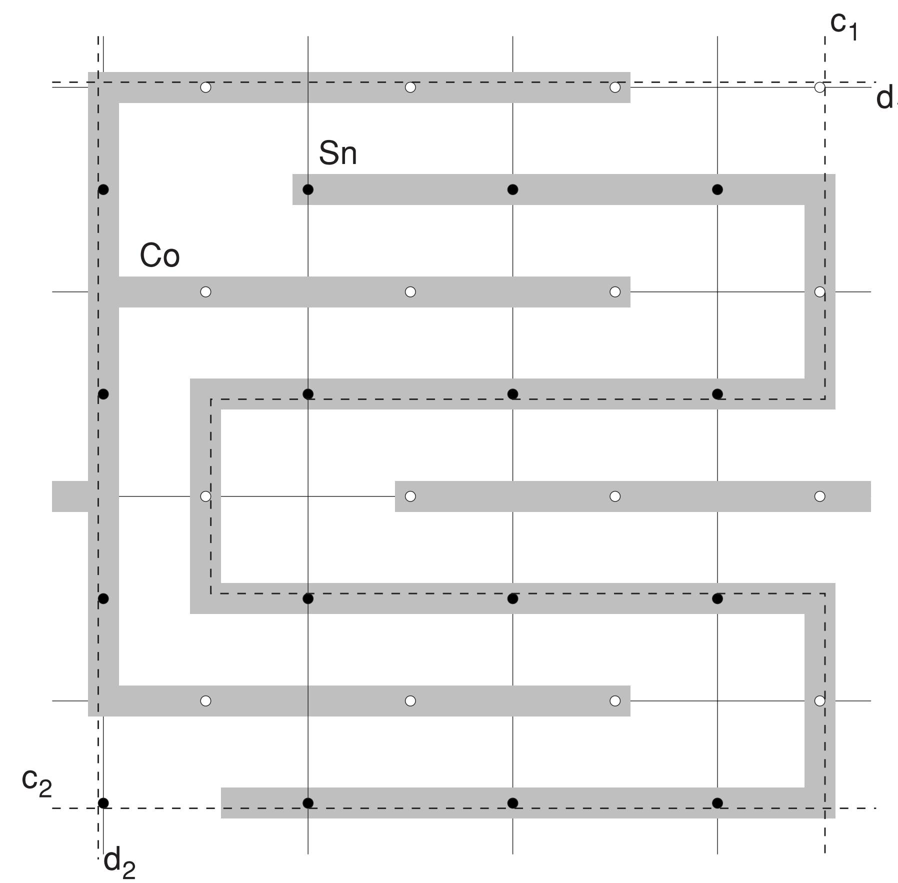

Spectral gap bound:

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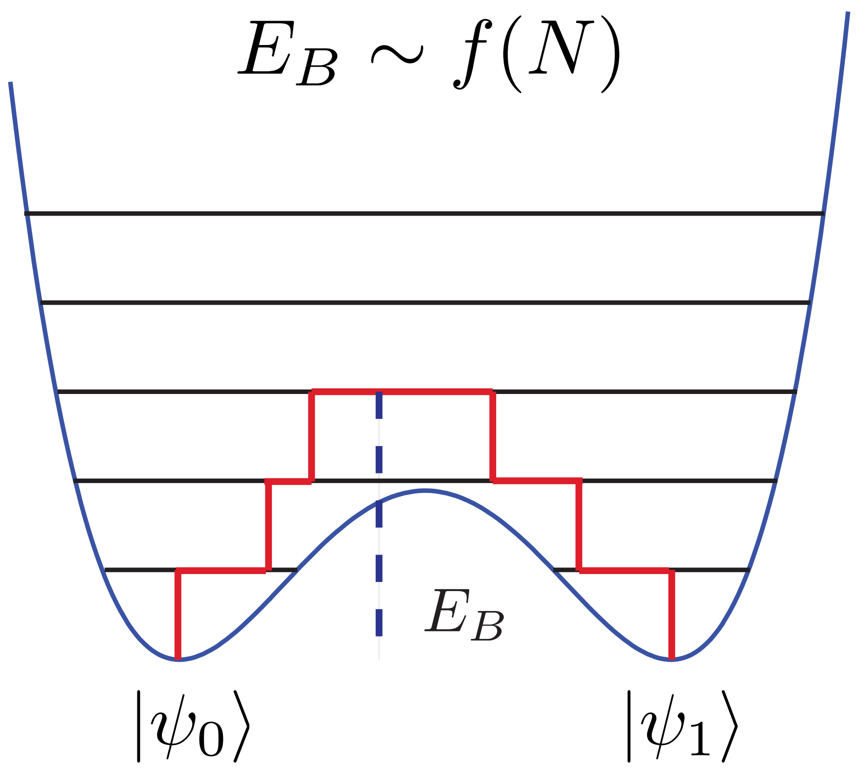
Implies mixing time bound:

$$t_{mix} \leq \mathcal{O}(N e^{8\beta J})$$

The energy barrier



The energy barrier

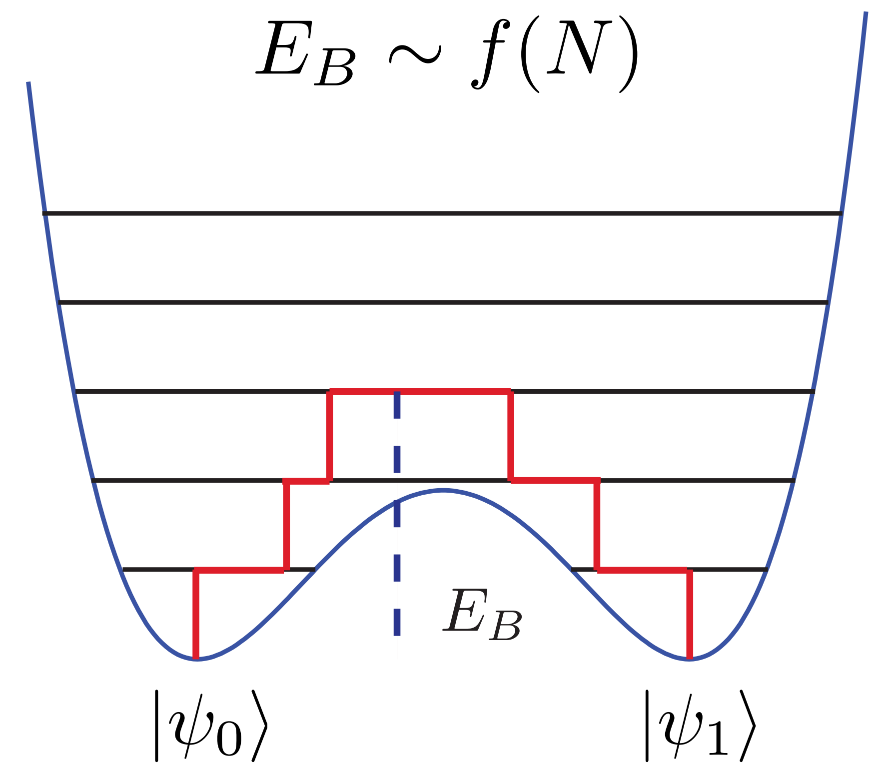
- Arrhenius law

$$t_{mem} \sim e^{\beta E_B}$$

Phenomenological law of the lifetime

Bravyi, Sergey, and Barbara Terhal, J. Phys. 11 (2009) 043029

Olivier Landon-Cardinal, David Poulin Phys. Rev. Lett. 110, 090502 (2013)



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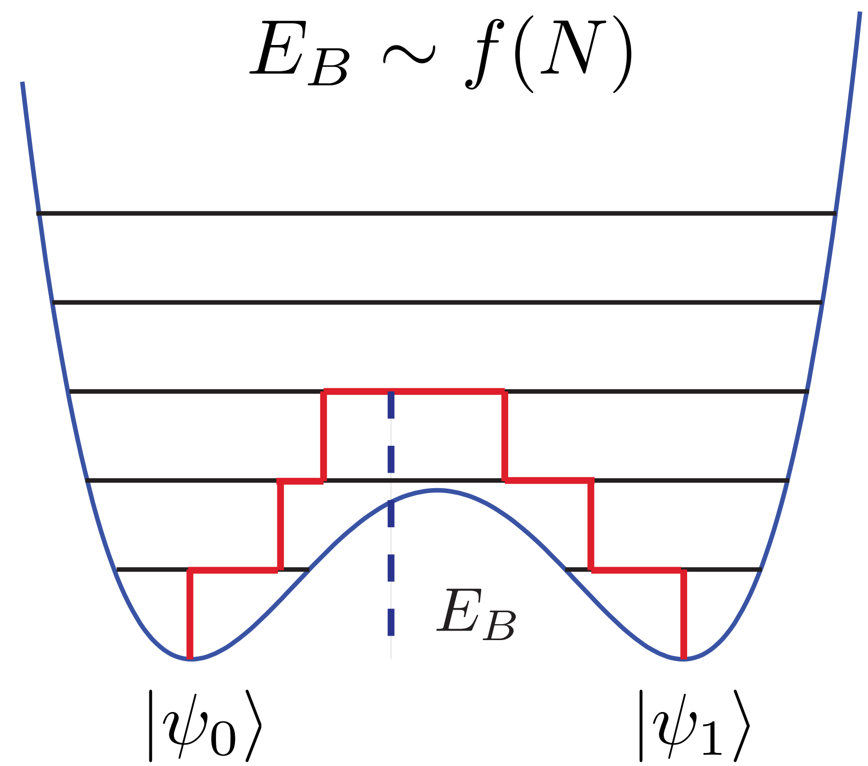


- Question:

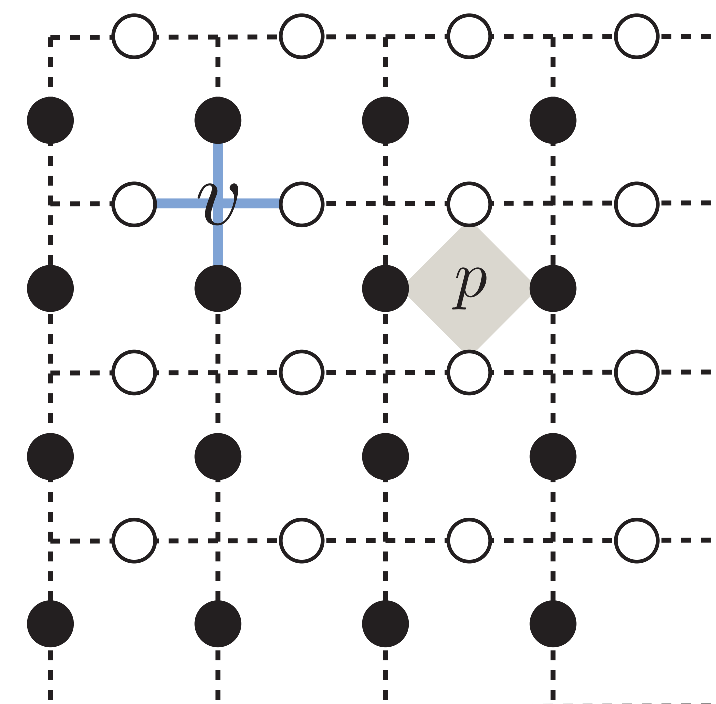
Can we prove a connection between the energy barrier and thermalization ?

Stabilizer Hamiltonians

A set of commuting Pauli matrices

$$\mathcal{G} = \{g_1, \ldots, g_M\} \qquad [g_i, g_j] = 0$$

Example : Toric Code



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$$B_p = X_1 X_2 X_3 X_4$$

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Logical operators $\mathcal{C}(\mathcal{S}) \backslash \mathcal{S}$

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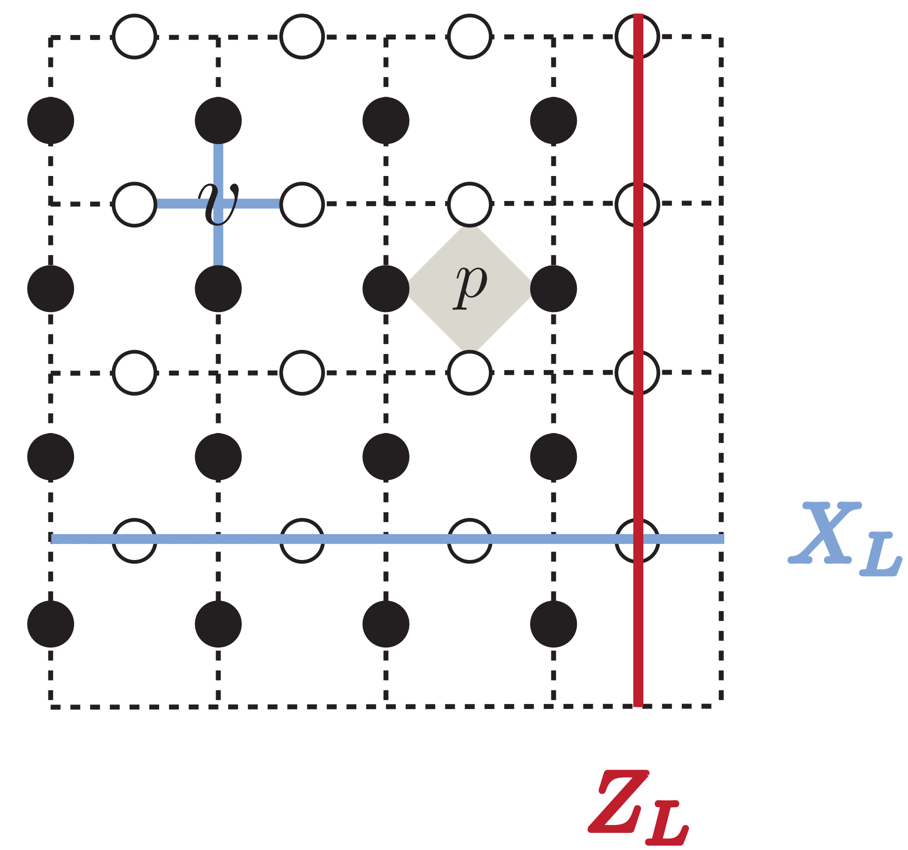

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Stabilizer Hamiltonian

$$H = -J \sum_k g_k$$

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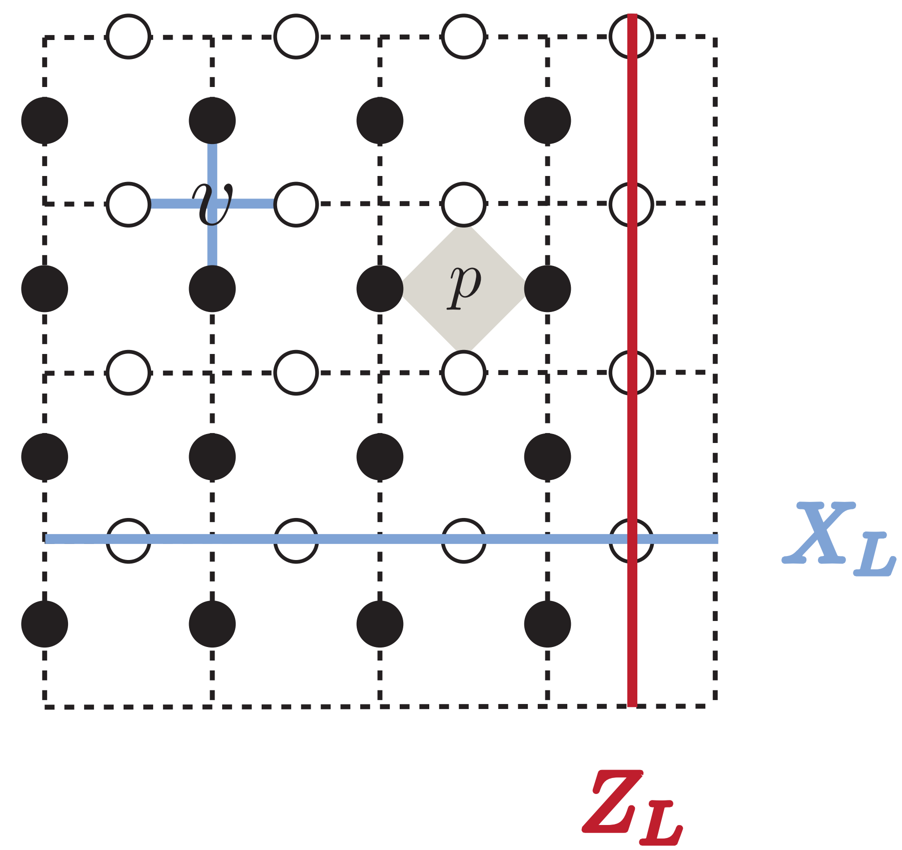

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Open system dynamics

- Lindblad master equation

$$\dot{\rho} = \mathcal{L}(\rho) = -i[H, \rho] + \sum_k L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}_+$$

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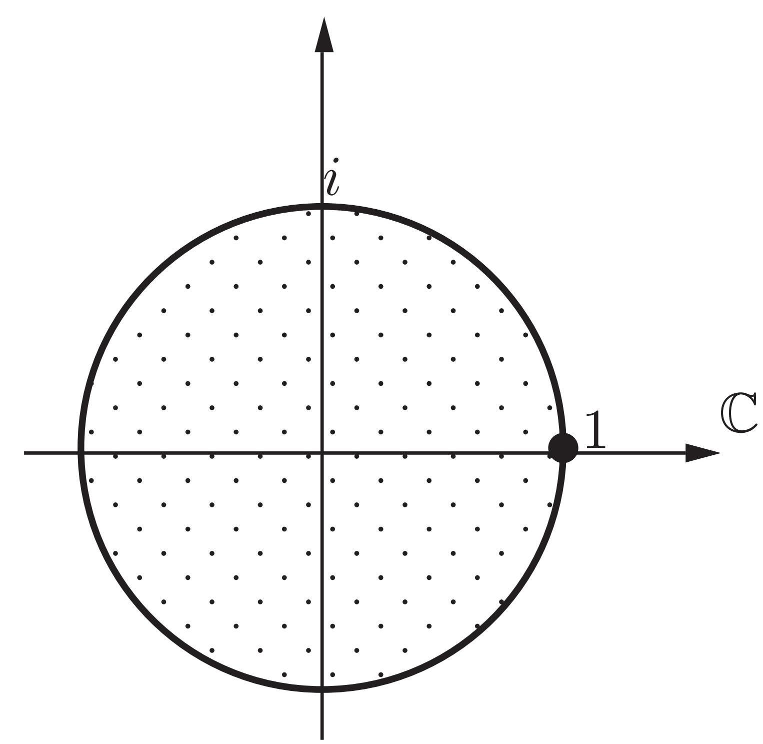
$$T_t(f) = \exp(\mathcal{L}t)(f)$$



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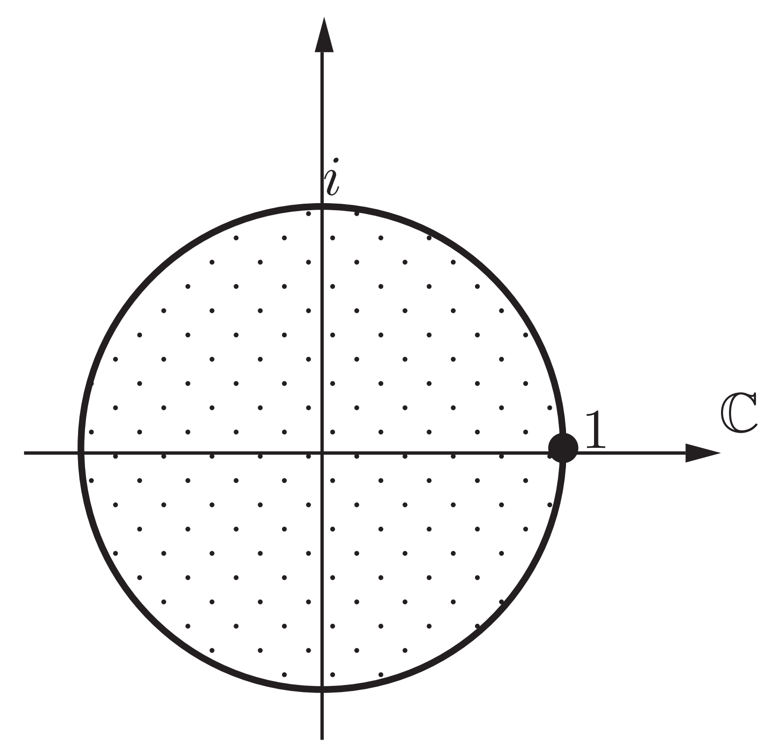
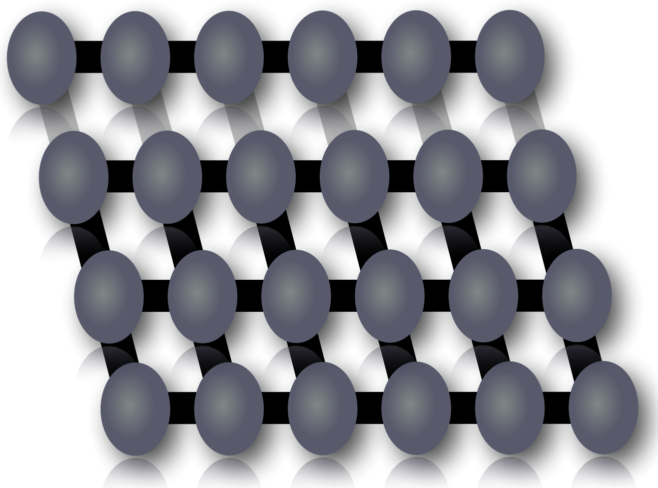


- With a unique fixed point

$$\mathcal{L}(\sigma) = 0 \qquad \sigma > 0$$

Thermal noise model & Weak coupling limit



H_S

$$H_I = \lambda \sum_\alpha S_\alpha \otimes B_\alpha$$

$\rho_R \propto e^{-\beta H_B}$

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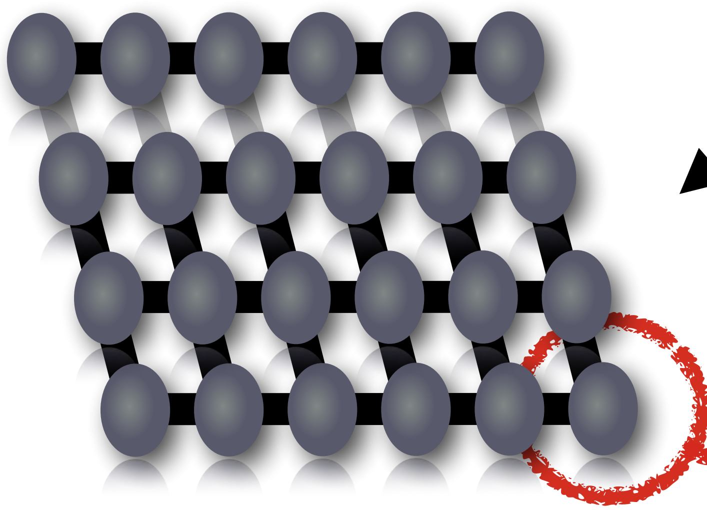

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$$S_\alpha = \sigma^x, \sigma^y, \sigma^z$$

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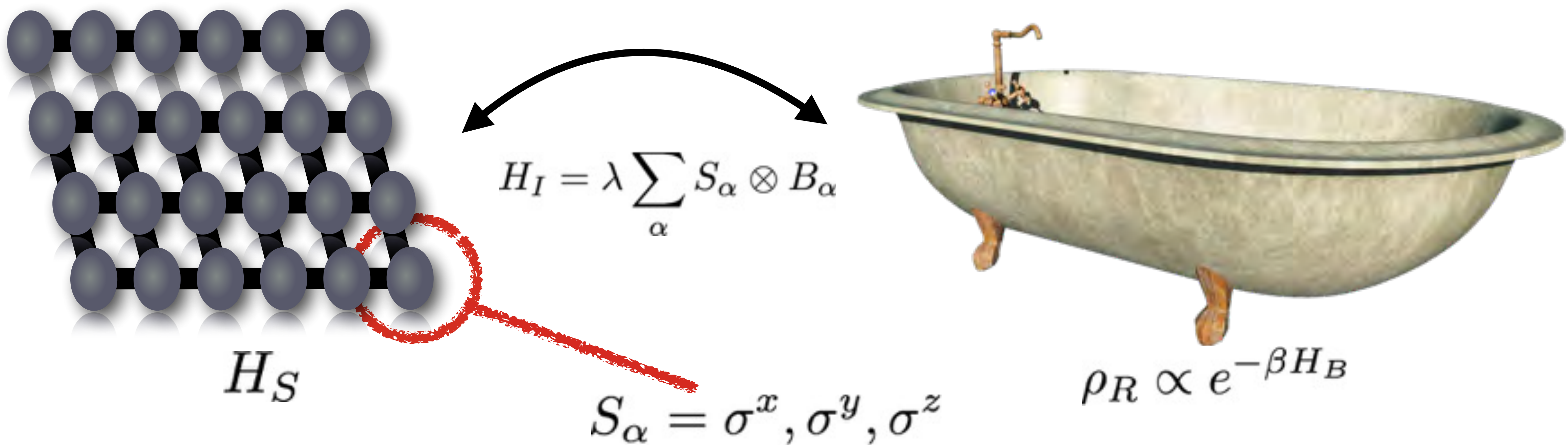

The evolution :

$$\rho_S(t+\Delta t) = \mathrm{tr}_R[e^{-iH\Delta t}(\rho(t)\otimes\rho_R)e^{iH\Delta t}]$$

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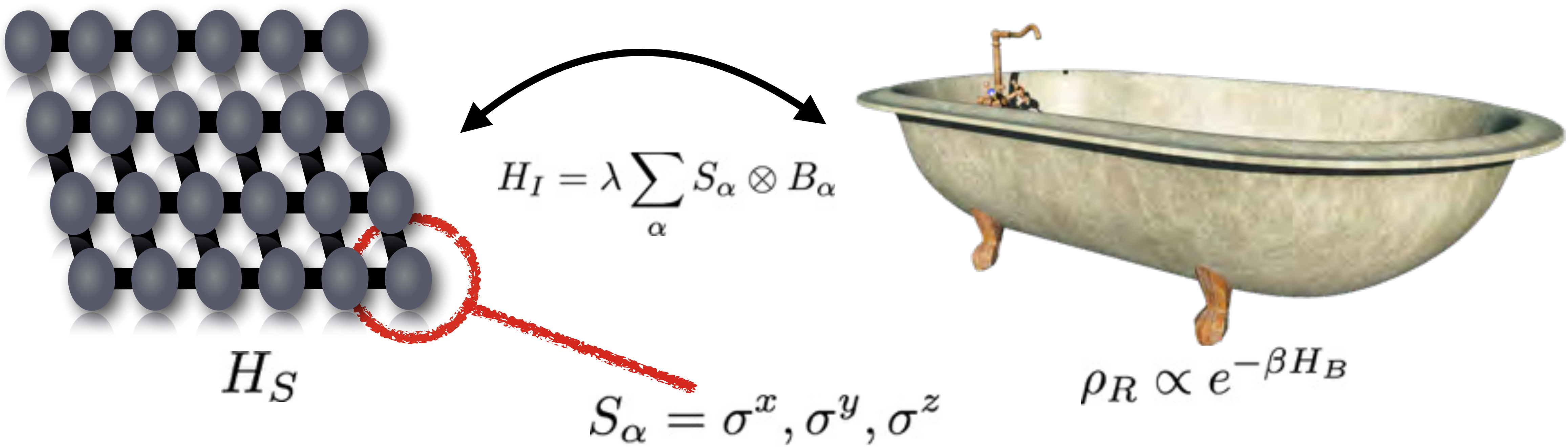

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Weak coupling limit & Markovian approximation:

$$\partial_t \rho = \mathcal{L}(\rho)$$

Davies, E. B. (1974). Markovian master equations.
Communications in Mathematical Physics, 39(2), 91–110.

The Davies generator

$$\mathcal{L}_\beta(\rho) = \sum_{\alpha,\omega} h^\alpha(\omega) \left(S_\alpha(\omega)\rho S_\alpha^\dagger(\omega) - \frac{1}{2}\{S_\alpha^\dagger(\omega)S_\alpha(\omega), \rho\} \right)$$

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For a single thermal bath:

KMS conditions*:

$$h^\alpha(-\omega) = e^{-\beta\omega} h^\alpha(\omega)$$

$$\sigma S_\alpha(\omega) = e^{\beta\omega} S_\alpha(\omega)\sigma$$

Ensures detail balance with:

$$\sigma \propto e^{-\beta H_S}$$

Gibbs state as steady state

* Kubo, R. (1957). Statistical-Mechanical Theory of Irreversible Processes. I. General Theory and Simple Applications to Magnetic and Conduction Problems. Journal of the Physical Society of Japan, 12(6), 570–586.
Martin, P., & Schwinger, J. (1959). Theory of Many-Particle Systems. I. Physical Review, 115(6), 1342–1373.

Davies generator for Pauli stabilizers

- Lindblad operators

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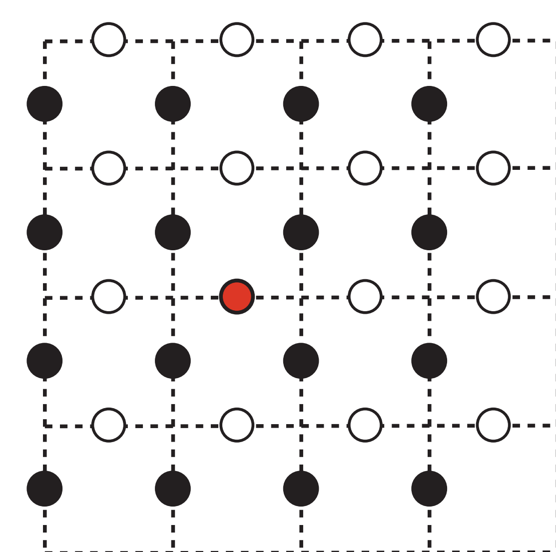

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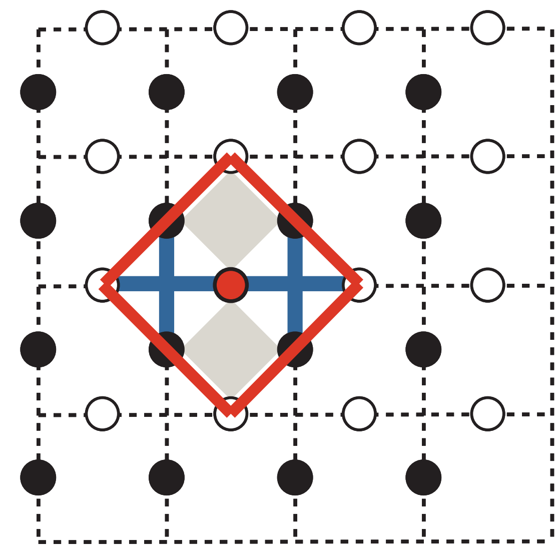

Convergence to the fixed point σ

- For a unique fixed point:

$$t > t_{\mathrm{mix}}(\epsilon) \quad \Rightarrow \quad \|e^{\mathcal{L}t}(\rho_0) - \sigma\|_{tr} \leq \epsilon$$

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- A thermal σ implies the bound

$$\|\sigma^{-1}\| \sim e^{c\beta N} \qquad \Longrightarrow \qquad t_{mix} \sim \mathcal{O}(\beta N \lambda^{-1})$$

Spectral gap bound

Theorem 14 *For any commuting Pauli Hamiltonian H, eqn. (1), the spectral gap λ of the Davies generator $\mathcal{L}_\beta$, c.f. eqn (15), with weight one Pauli couplings W_1 is bounded by*

$$\lambda \geq \frac{h^*}{4\eta^*} \exp(-2\beta\, \overline{\epsilon}), \tag{81}$$

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The constants are:

The largest Pauli path: $\eta^* = \mathcal{O}(N)$

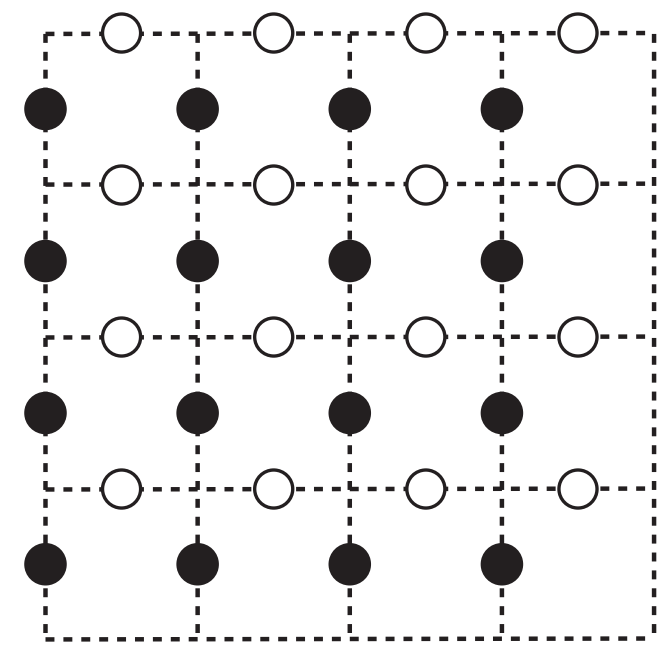
smallest transition rate: $h^* \geq c_0 e^{-\beta\Delta}$

generalized energy barrier : $\overline{\epsilon}$

Generalized energy barrier

Paths on the Pauli Group $\gamma_0, \gamma_1, \ldots, \gamma_t \in \mathcal{P}$

$\gamma_0 = I$



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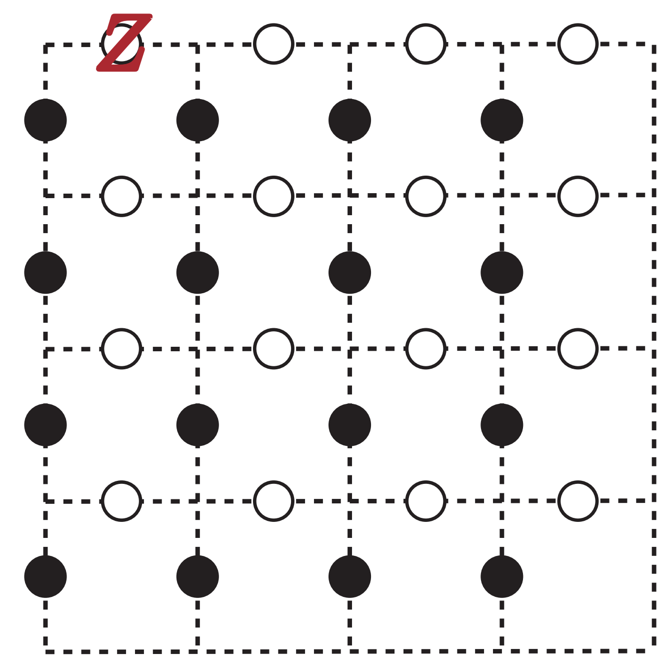

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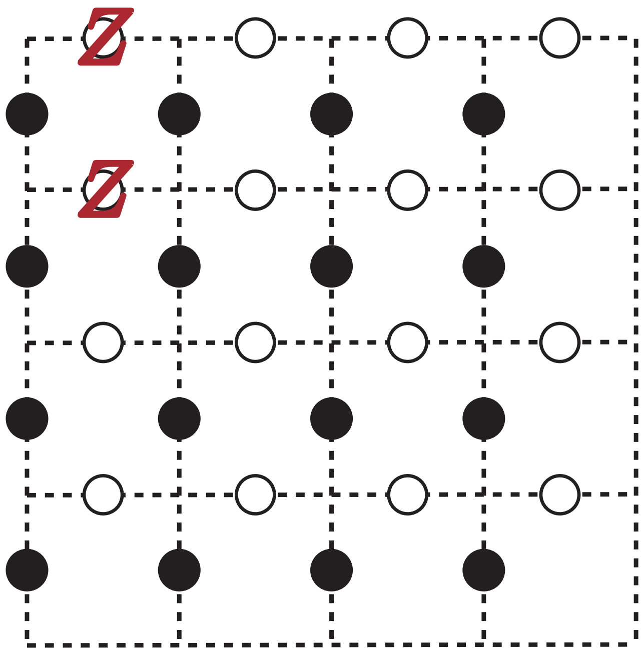

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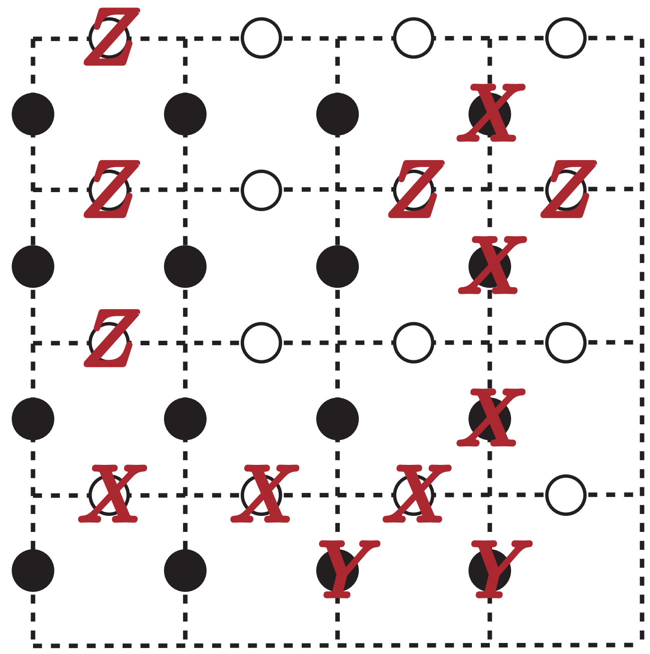

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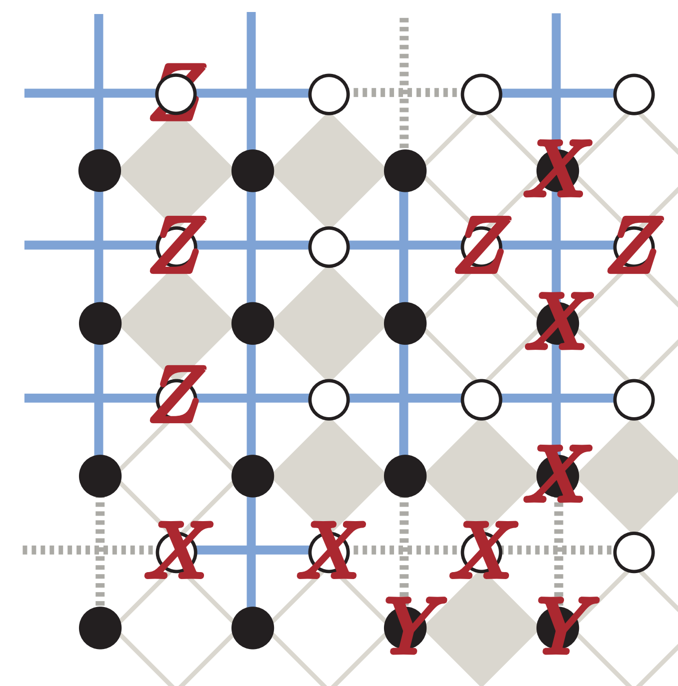

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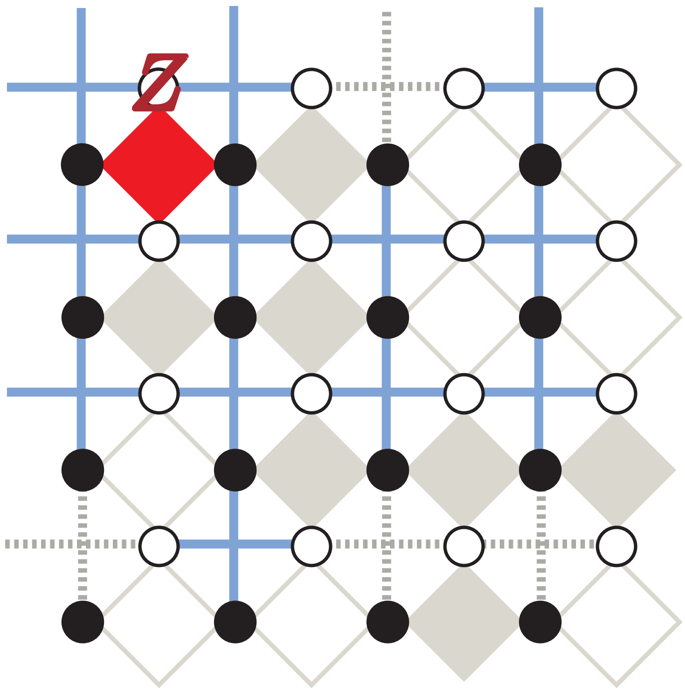

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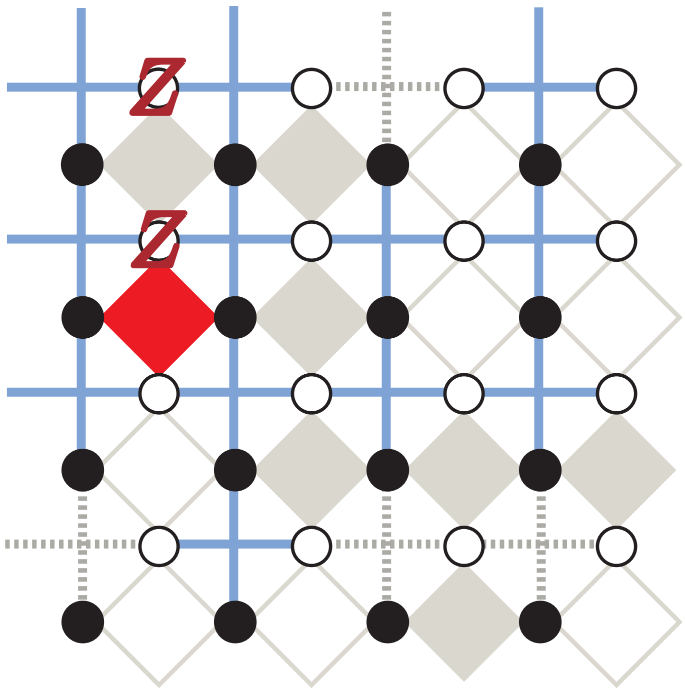

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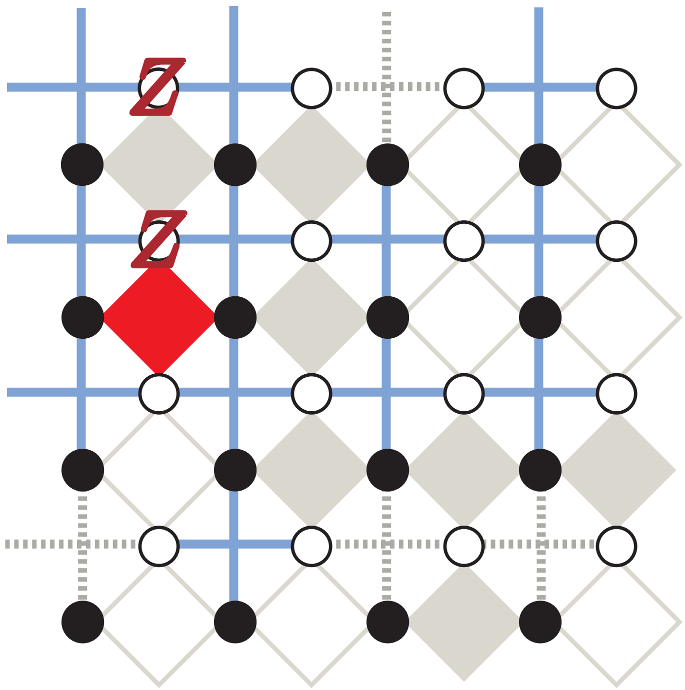

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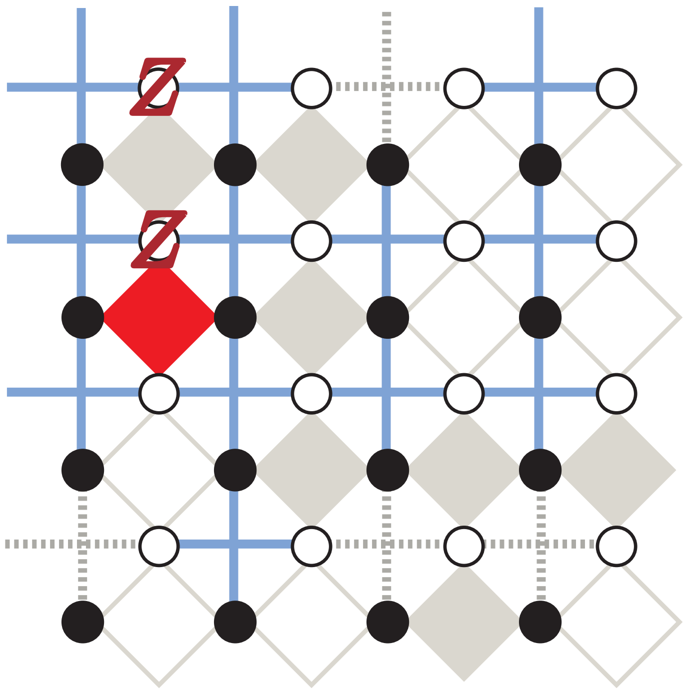

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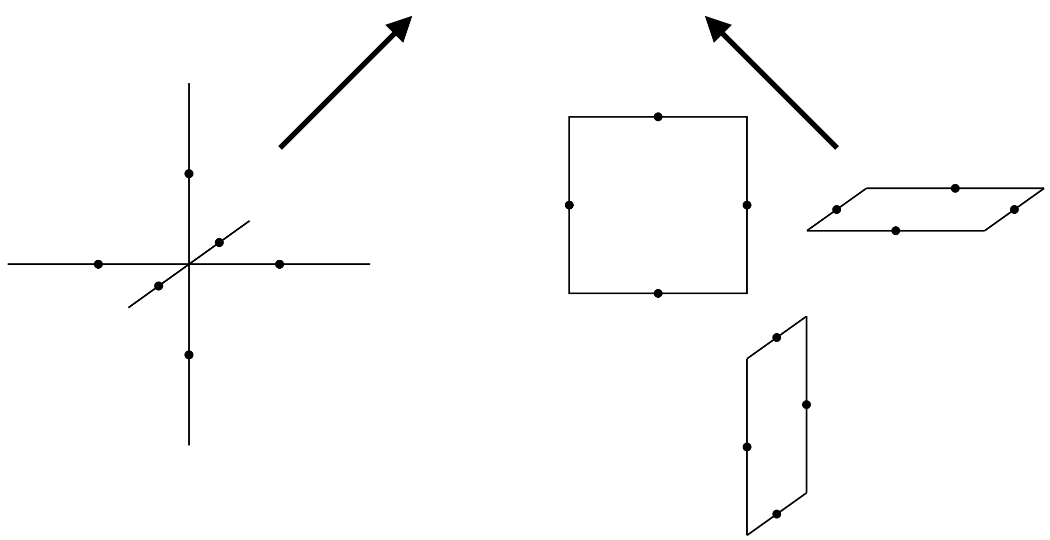
Example: 2D Toric Code

$$\lambda \geq \frac{1}{8N} e^{-\beta 6J}$$

3D Toric Code

Consider the toric code on an $L \times L \times L$ lattice

$$H = -J \sum_v A_v - J \sum_p B_p$$



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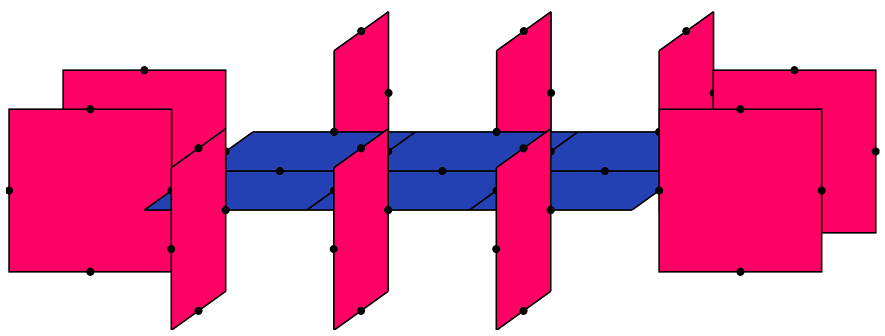

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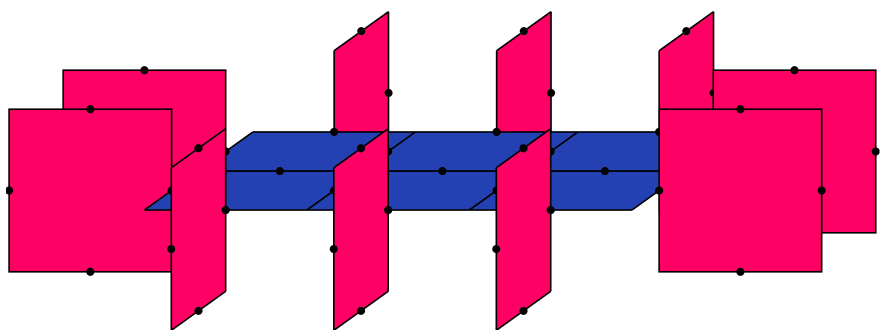

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3D Toric Code

Fernando Pastawski

Michael Kastoryano

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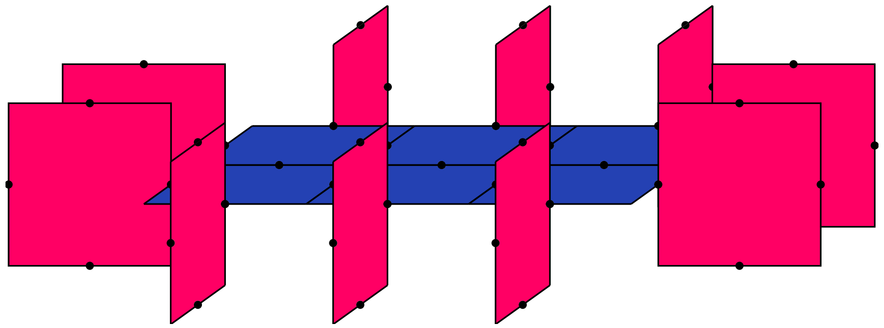

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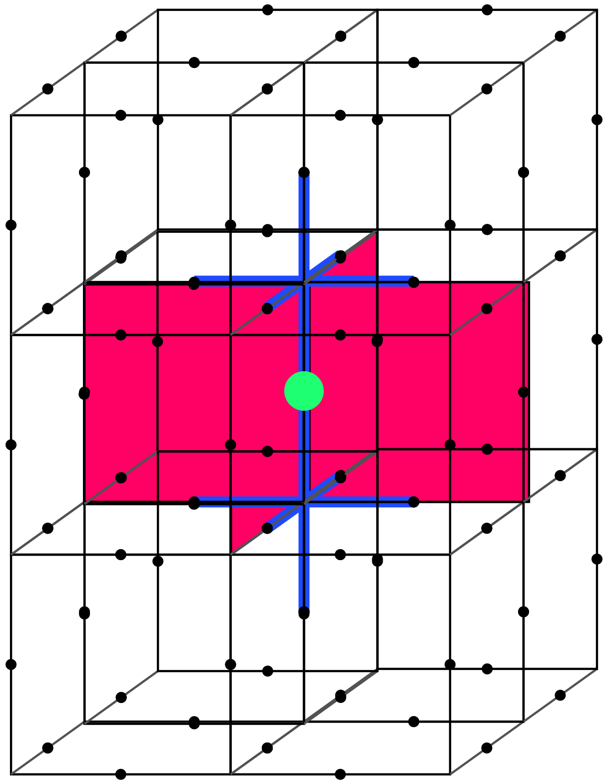
$\kappa(\beta)$



High temperature bound

$$\kappa(\beta^*) \leq 1 \qquad \lambda \geq \frac{1-\kappa(\beta)}{\log(2)}$$

Discussion of the bound

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- It would be nicer to have a bound that includes "entropic contributions"

- Can we get rid of the 1/N factor?

$$\lambda \geq \frac{h^*}{4N}\, e^{-2\beta\bar{\epsilon}}$$

Proof sketch

- The Poincare Inequality
- Matrix pencils and the PI
- The canonical paths bound
- The spectral gap and the energy barrier

The Poincare Inequality

$$\lambda \mathrm{Var}_\sigma(f,f) \leq \mathcal{E}(f,f)$$

The Poincare Inequality

$$\lambda\left(\mathrm{tr}\left[\sigma f^\dagger f\right] - \mathrm{tr}\left[\sigma f\right]^2\right) \leq -\mathrm{tr}\left[\sigma f^\dagger \mathcal{L}(f)\right]$$

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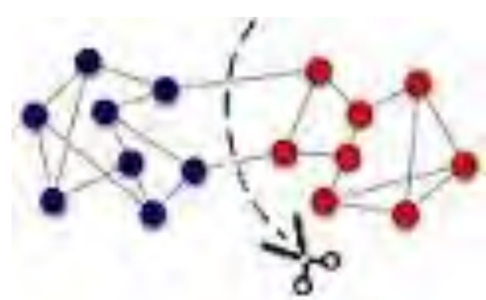
For classical Markov processes

- Sampling the Permanent : M. Jerrum, A. Sinclair. "Approximating the permanent." *SIAM journal on computing* 18.6 (1989): 1149-1178.
- Powerful because it can lead to a geometric interpretation



Cheeger's bound

Canonical paths

The Poincare Inequality

MIND THE GAP

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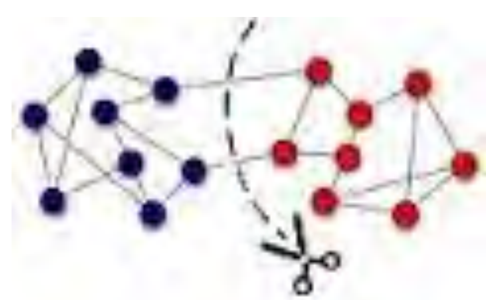

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Challenges in the quantum setting

- We are missing a general geometric picture

Poincare and a Matrix pencil $\lambda^{-1} = \tau$

Equivalent formulation for $\lambda \mathrm{Var}_\sigma(f,f) \leq \mathcal{E}(f,f)$

minimize τ subject to $\tau \hat{\mathcal{E}} - \hat{\mathcal{V}} \geq 0$

where $\mathcal{E}(f,f) = (f|\hat{\mathcal{E}}|f)$ and $\mathrm{Var}_\sigma(f,f) = (f|\hat{\mathcal{V}}|f)$

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Lemma: Let $\hat{\mathcal{E}} = AA^\dagger$ and $\hat{\mathcal{V}} = BB^\dagger$

$$\tau = \min ||W||^2 \quad \text{subject to} \quad AW = B$$

Boman, Erik G., and Bruce Hendrickson. "Support theory for preconditioning." SIAM Journal on Matrix Analysis and Applications 25.3 (2003): 694-717.

Suitable matrix factorization

Some intuition from $\beta \to 0$

$$\mathcal{E}(f,f) \longrightarrow \mathcal{L}(f) \sim \sum_{i:\alpha_i} (\sigma_i^{\alpha_i} f \sigma_i^{\alpha_i} - f)$$

Suitable matrix factorization

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Choosing a decomposition in terms of

$$(\sigma_1^x f \sigma_1^x - f) + (\sigma_2^z f \sigma_2^z - f) + (\sigma_3^x f \sigma_3^x - f) \quad \sim \quad (\sigma_1^x \sigma_2^z \sigma_3^x \; f \; \sigma_1^x \sigma_2^z \sigma_3^x - f)$$

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A generalization yields to the matrix triple $[A, B, W]$

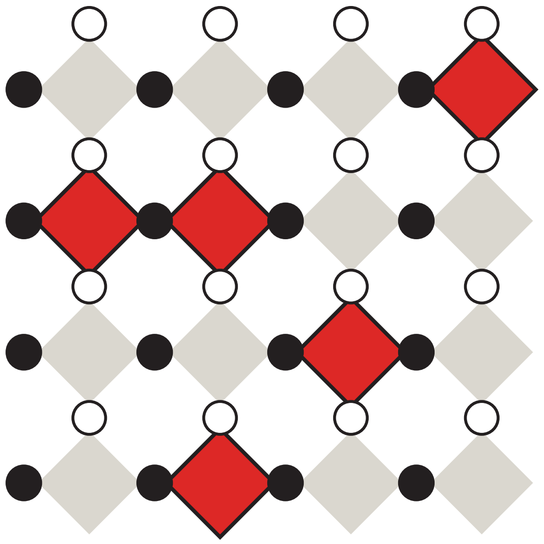
$\|W\|^2$ can be bounded by suitable norm bounds

Canonical paths bound

- The norm bound on $\|W\|^2$ can be evaluated in the following picture

Dressed Pauli paths :

$$\hat{\eta_a} = [(a, \mathbf{0}), (a^{\alpha_1}, \alpha_1), \ldots, (a^{\eta}, \eta)]$$



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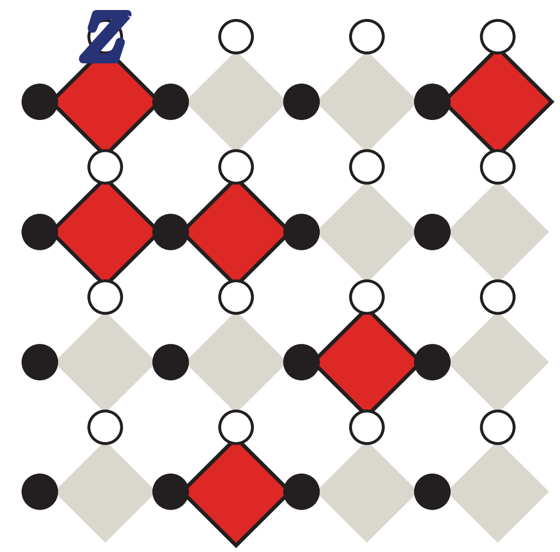

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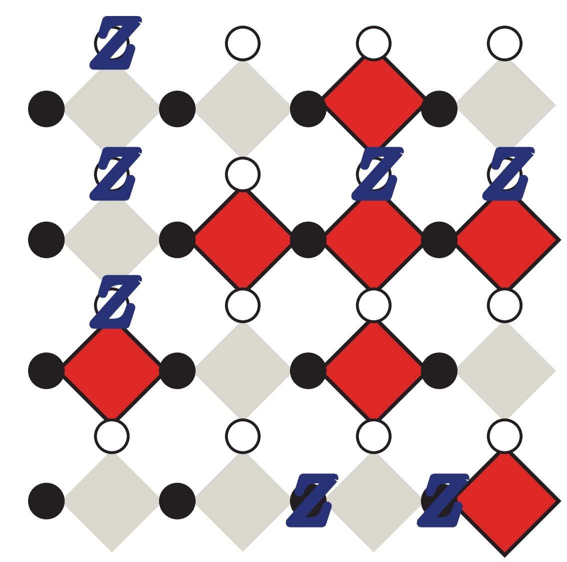

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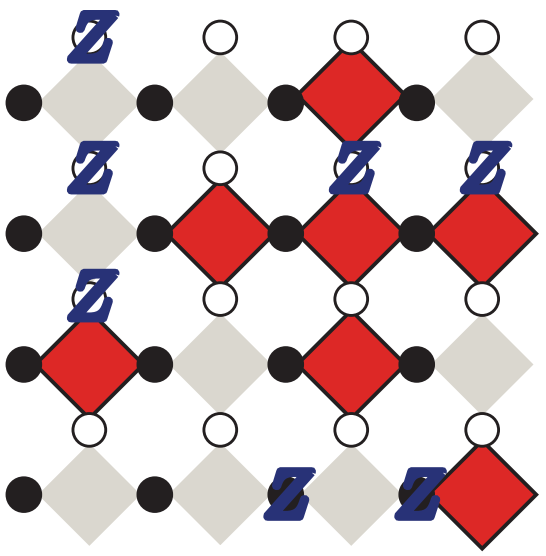

The matrix norm bound yields

$$\tau \leq \max_{\xi} \frac{4\eta^*}{2^N h(\omega^{\alpha}(b))\rho_b} \sum_{\hat{\eta}_a \in \Gamma(\xi)} \rho_a \rho_{a^{\eta}}$$

The spectral gap and the energy barrier

- The only challenge is the maximum in the definition of τ

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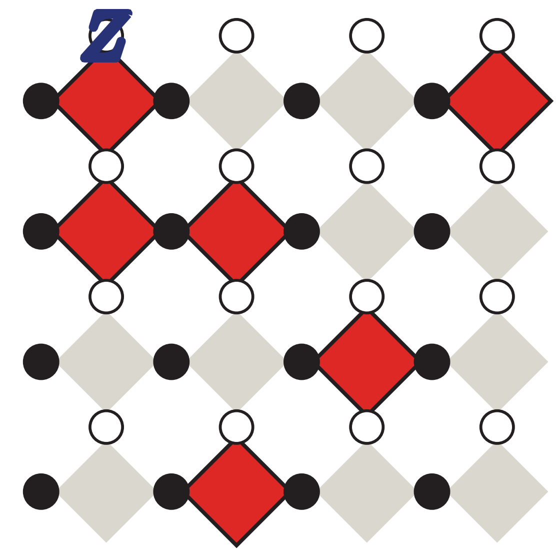
Injective map (Jerrum & Sinclair)

$$\Phi_\xi : \Gamma(\xi) \to \mathcal{P}_N$$



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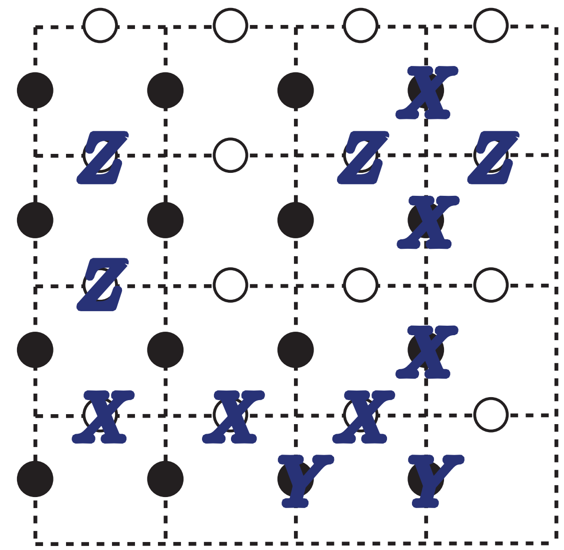

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$$\Phi_\xi : \Gamma(\xi) \to \mathcal{P}_N \qquad [\Phi_\xi(\hat{\eta}_a)]_k = \begin{cases} (0,0)_k & : k \leq \xi \\ \eta_k & : k > \xi \end{cases}$$

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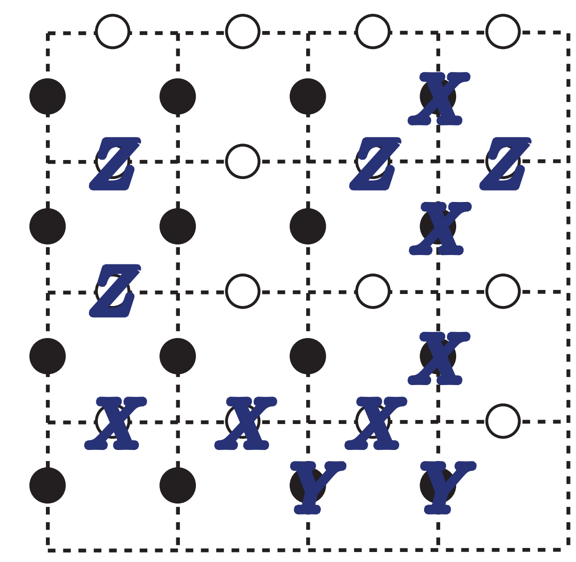

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Bounding $\mathcal{T}$

$$\epsilon_{b^{\eta \oplus \xi}} + \epsilon_{b^\xi} - \epsilon_b - \epsilon_{b^\eta} \leq 2\bar{\epsilon}$$

The spectral gap and the energy barrier

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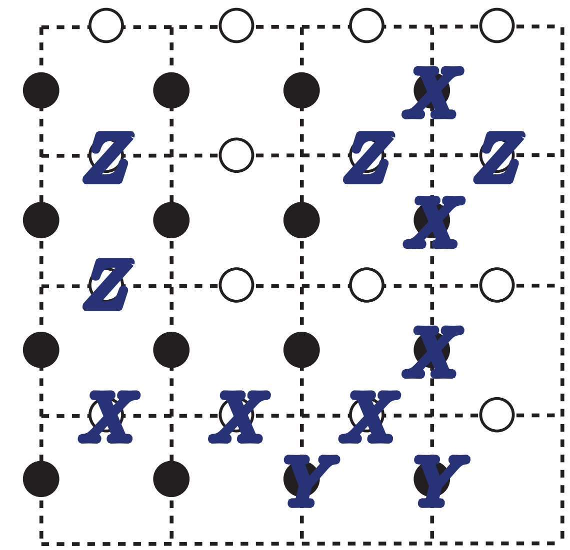

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$$h^\alpha(\omega^\alpha(a))\rho_a \rho_{b^{\Phi_\xi(\hat{\eta}_b)}} \geq h^* e^{-\beta 2 \overline{\epsilon}} \rho_b \rho_{b^\eta}$$

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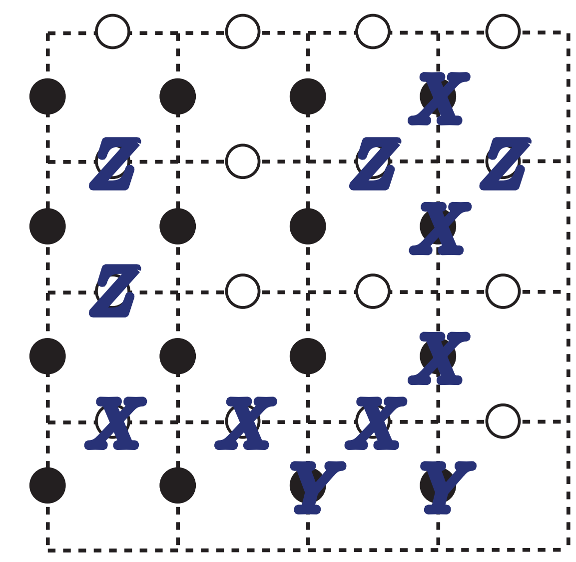

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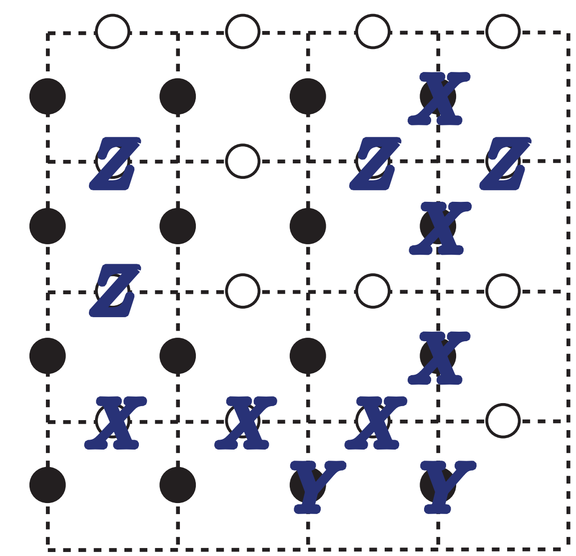

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Conclusion and Open Questions

- Is it possible to find a bound that also takes the “entropic” contributions into account?
- Can we get rid of the prefactor? N^{-1}
- It would be great if one could extend the results to more general quantum memory models.
- This only provides a converse to the lifetime of the classical memory. It would be great if one could find a converse for the quantum memory time


THANK
YOU