Cellular-automaton decoders for topological quantum memories

arXiv:1406.2338

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Outline

- ϕ-Automaton Decoders
  - 2D*-decoder
  - 3D-decoder
- Dynamic Setting
- Outlook & Conclusion

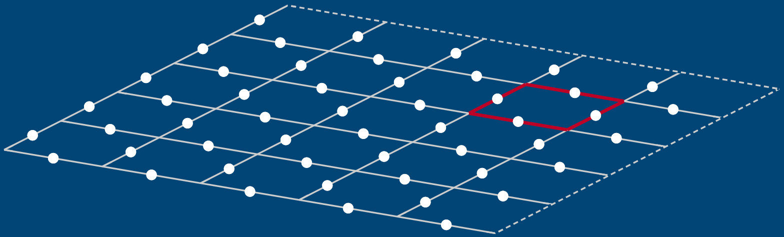
New decoders for the 2D toric code?

- Sophisticated decoders exist
  - Realspace RG Decoder $O(\log L)$ [1]
  - MWPM $O(L^2)$ [2]
- Parallelization does not imply locality
  - Hidden communication costs
  - Not necessarily suited for embedded hardware
- Question we address
  - Natural parallelization without hidden costs
  - Connecting decoding with physical systems

[1] G. Duclos-Cianci and D. Poulin, Phys. Rev. Lett. **104**, 050504 (2010)
[2] A. G. Fowler, A. C. Whiteside, and L. C. L. Hollenberg, Phys. Rev. Lett. **108**, 180501 (2012)

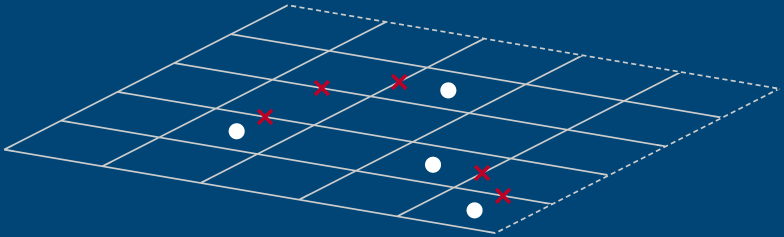
ϕ-Automaton Decoders

2D toric code



- Consider X-errors with probability p
- Consider plaquette operators ($Z^{\square}$)

Setting: fields for the anyons



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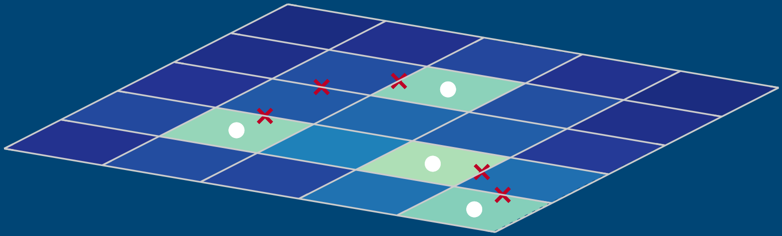

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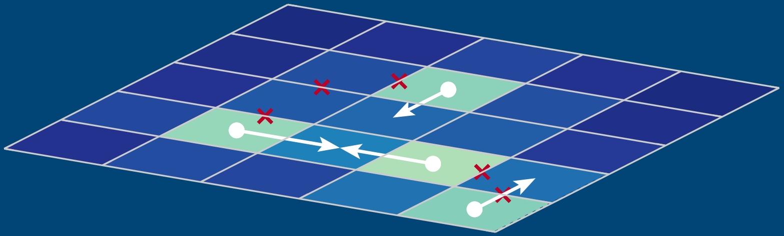

- Implementation as classical cellular automaton?

Local field generation

Differential equation $\nabla^2 \Phi(\mathbf{r}) = q(\mathbf{r})$

discretization

$\mathbf{r} \in \mathbb{R}^D \rightsquigarrow \mathbf{x} \in \mathbb{Z}^D$

Set of linear equations $-2D\phi(\mathbf{x}) + \sum_{\langle \mathbf{y},\mathbf{x} \rangle} \phi(\mathbf{y}) = q(\mathbf{x})$

Jacobi method

$\phi(\mathbf{x}) \rightsquigarrow \phi_{t+1}(\mathbf{x})$

$\phi(\mathbf{y}) \rightsquigarrow \phi_t(\mathbf{y})$

Interation $\phi_{t+1}(\mathbf{x}) = \operatorname{avg}_{\langle \mathbf{y},\mathbf{x} \rangle} \phi_t(\mathbf{y}) + q(\mathbf{x})$

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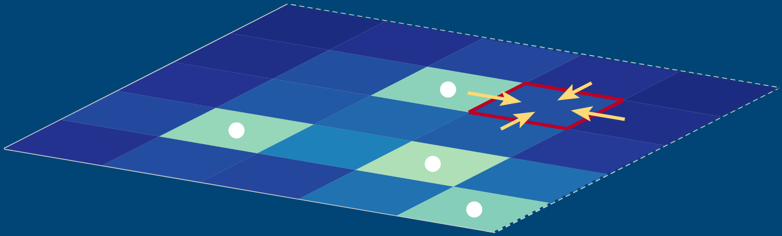
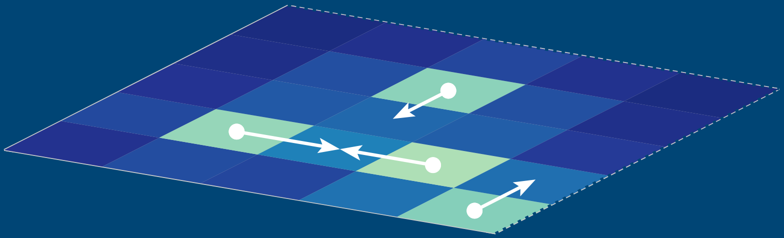
1. Field-updates (ϕ-automaton)



- Every cell stores a field value $\phi(x)$
- Update rule: Average of neighboring fields + charge

2. Anyon-updates



- Anyons *move* via X-flips on crossed edges
- Update rule: Move to the neighbor cell with maximal ϕ-value

The decoding algorithm

Sequence
- $c\times$ field-update
- $1\times$ anyon-update

- Algorithm: Repeat *sequence* until all anyons are fused

Field velocity c Parameter for the ‘speed of field propagation’

The 2D*-decoder

Sequence*

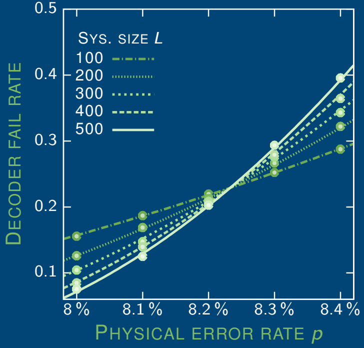
- $c\times$ field-update
- $1\times$ anyon-update
- $\boxed{c \to c + 0.2}$

- Algorithm: Repeat *sequence** until all anyons are fused

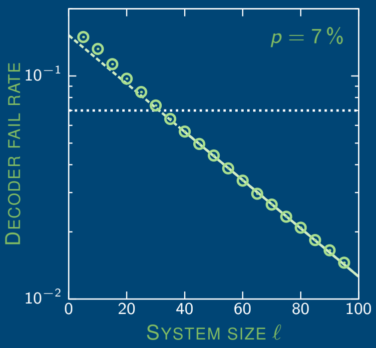
Field velocity c Parameter for the 'speed of field propagation'

2D*-decoder: numerical analysis

Recovery threshold 8.2 %



Exponential suppression



- Runtime: $O(\log^{7.5} L)$
- Resembles Wootton's decoder [3]

[3] J. R. Wootton, *A simple decoder for topological codes*, `arXiv:1310.2393`

Avoiding sequence dependence

- Finding the right field
  - $c \to \infty$ must not be a problem
  - Before: Poisson's equation in 2D
    - $-\log r$ profile
  - Idea: Try fields of the form

$$\phi(r) \sim \frac{1}{r^\alpha}$$

- Simulation with explicit fields, **no cellular automaton**
  - Anyons move towards max. field (as before)

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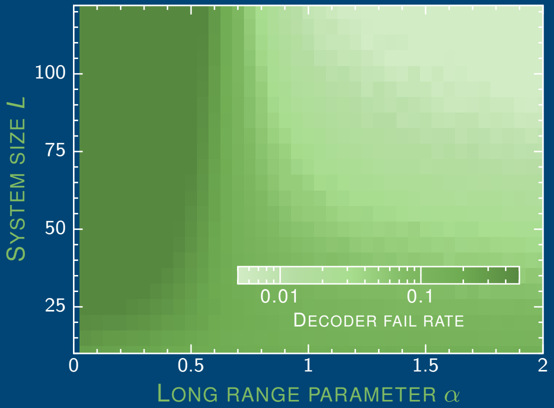
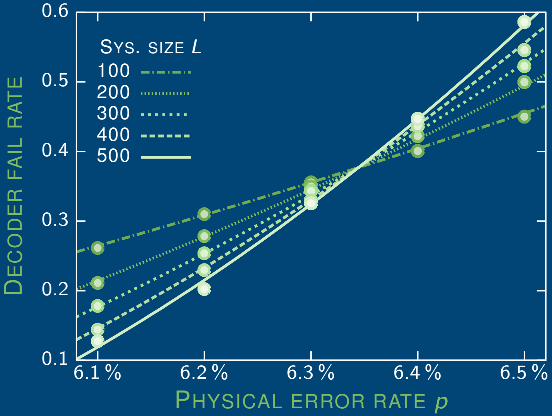
- Conjecture: At $\alpha = 1$, transition from not-decoding to decoding regime

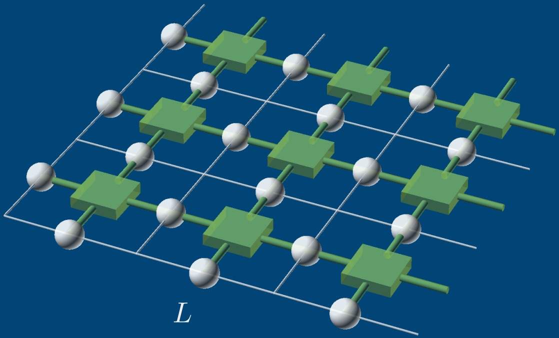


Avoiding sequence dependence

- Conjecture: At $\alpha = 1$, transition from not-decoding to decoding regime
- $\Rightarrow \phi(r) \sim 1/r$ should work as decoder



3D ϕ-automaton



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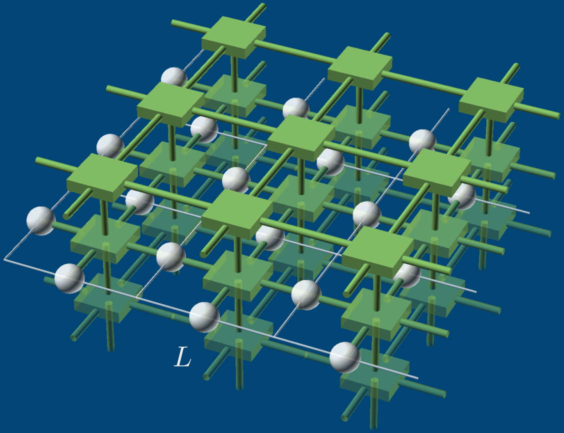
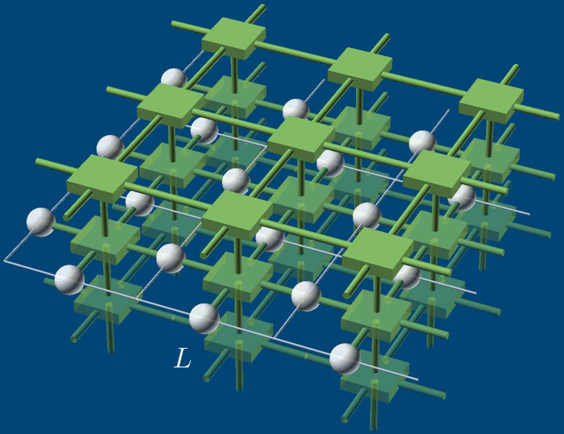


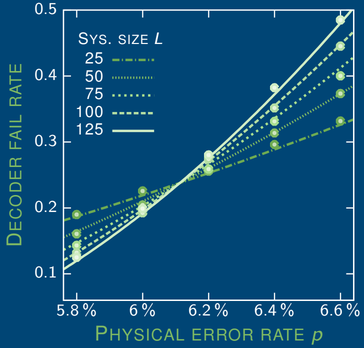
3D ϕ-automaton

- Sufficient field convergence if $c(L) \sim \log^2 L$



3D decoder: numerical analysis

Recovery threshold: 6.1 %



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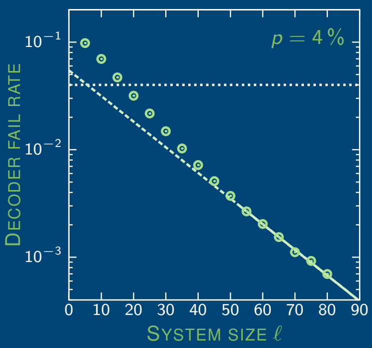
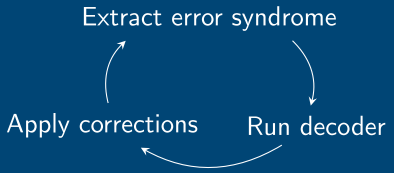


- Runtime: $O(\log^3 L)$
- Fundamentally new working principle

Dynamic Setting

Static setting



Dynamic setting

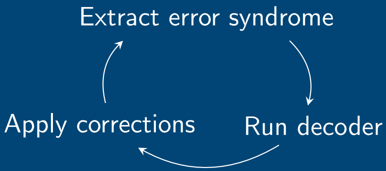
Extract error syndrome $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\cdots$

Run decoder $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\cdots$

Apply corrections $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $\cdots$

- Decoder has to take new information into account
- 3D decoder has no ‘time zero’
  - Promising candidate to work in dynamic setting

Static setting



Dynamic setting

Extract error syndrome $\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \cdots$
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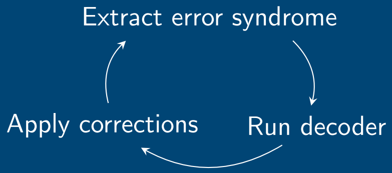

Dynamic setting

Extract error syndrome	$\longrightarrow$			$\longrightarrow$			$\longrightarrow$	$\cdots$
Field-updates	$\longrightarrow$	$\longrightarrow$	$\longrightarrow$	$\longrightarrow$	$\longrightarrow$	$\longrightarrow$	$\longrightarrow$	$\cdots$
Anyon-updates	$\longrightarrow$			$\longrightarrow$			$\longrightarrow$	$\cdots$

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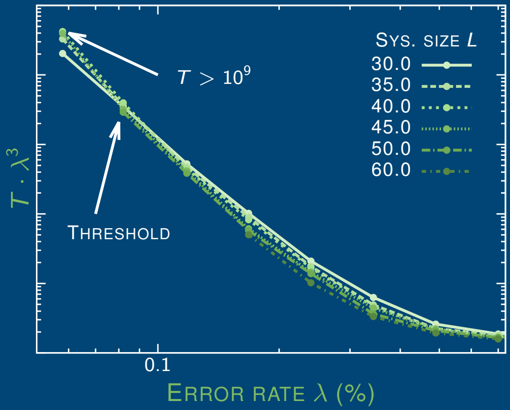
Numerical results



Further results

- Measurement errors only shift threshold
- Classical hardware can be imperfect
- No waiting for measurement outcomes

Outlook & Conclusion

Open questions

- Can the overhead $c(L) \sim \log^2 L$ be avoided?
  - Harrington: $\log L$ layers of hardware [4]
  - Gács': purely constant overhead (self-simulation and complicated update rules) [5]
  - Hardware complexity for constant overhead: unknown
- Other error correcting codes and error models
- Dissipative self correcting memories
- Relation to Toom's stability theorem [6]

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[4] J. W. Harrington, *Analysis of quantum error-correcting codes: symplectic lattice codes and toric codes*, PhD thesis (2004)

[5] P. Gács, J. Stat. Phys. **103**, 45 (2001)

[6] A. L. Toom, in Multicomponent random systems, Vol. 6, Advances in probability and related topics (1980), pp. 549–576.

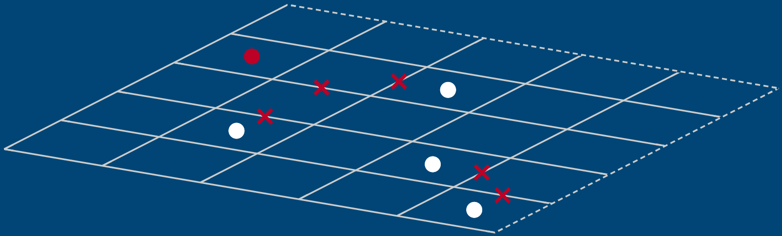
Conclusion

- Two local ϕ-automaton decoders
  - 2D*-decoder with threshold above 8.2 %
  - 3D-decoder with threshold above 6.1 %
- Entirely new working principle for decoders
- No hidden communication costs
- Simple wiring and suited for hardware implementation
- 3D-decoder can operate in the dynamic setting
  - Measurement errors are corrected
  - No strict requirement of synchronization
  - Handling probabilistic stabilizer measurements

Thank you for your attention!

Questions?

Measurement errors



- Measurements errors give an effective error rate λ
  - No specialized algorithm requited

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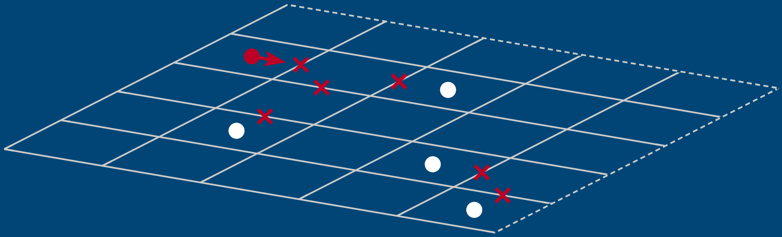

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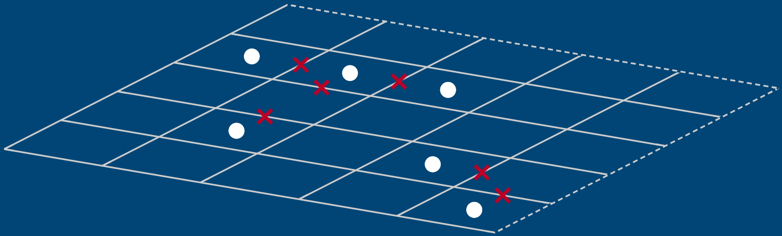

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