Quantum Error Correction for Long-Distance Quantum Communication

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Peter van Loock, Fabian Ewert, Marcel Bergmann

Overview

- ✓ Old versus New Quantum Repeaters: QED vs. QEC
- ✓ Photon Loss Codes
- ✓ Ultrafast Long-Distance Quantum Communication

Overview

- ✓ Old versus New Quantum Repeaters: QED vs. QEC
- ✓ Photon Loss Codes
- ✓ Ultrafast Long-Distance Quantum Communication with Linear Optics

Classification of Quantum Repeaters

1.) Original Quantum Repeaters (Briegel et al., DLCZ,…):

use entanglement distribution, swapping, purification
(loss, local errors)

2.) Quantum repeaters with purification (loss) and QECC (local errors)

3.) Quantum repeaters with QECC only (loss and local errors)

S. Muralidharan, J. Kim, N. Lütkenhaus, M.D. Lukin, and L. Jiang , PRL **112**, 250501 (2014)

Original Quantum Repeaters:
Quantum Error Detection
for Long-Distance
Quantum Communication

Direct Transmission of Flying Qubits

$$\left|\psi_{\text{in}}\right\rangle = \alpha\left|0\right\rangle + \beta\left|1\right\rangle$$

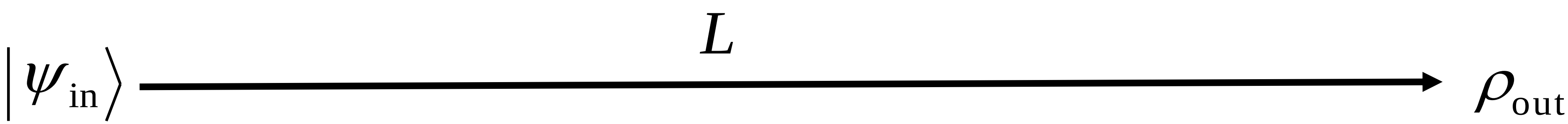
$$\left|\psi_{\text{in}}\right\rangle \xrightarrow{\quad L \quad} \rho_{\text{out}}$$

$$\rho_{\text{out}} = \left(\alpha\left|0\right\rangle + \beta\sqrt{\eta}\left|1\right\rangle\right)\times \text{H.c.} + \left|\beta\right|^2\left(1-\eta\right)\left|0\right\rangle\left\langle 0\right|$$

$$\eta = \exp\left(-L/L_{\text{att}}\right)$$

Direct Transmission of Flying Qubits

$$|\psi_{\text{in}}\rangle = \alpha|10\rangle + \beta\,|01\rangle$$

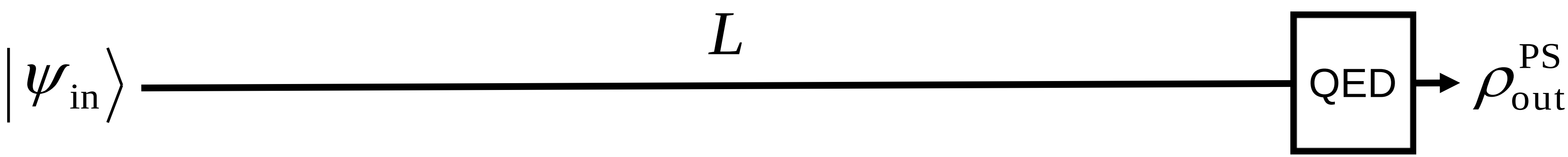


$$\rho_{\text{out}} = \eta\,|\psi_{\text{in}}\rangle\langle\psi_{\text{in}}| + (1-\eta)|00\rangle\langle 00|$$

$$\eta = \exp(-L/L_{\text{att}}) = F = \langle\psi_{\text{in}}|\rho_{\text{out}}|\psi_{\text{in}}\rangle$$

Direct Transmission of Flying Qubits

$$|\psi_{\text{in}}\rangle = \alpha|10\rangle + \beta|01\rangle$$

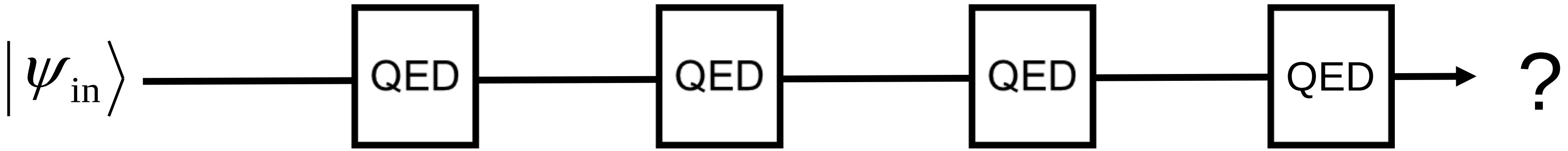


$$\rho_{\text{out}}^{\text{PS}} = \eta\,|\psi_{\text{in}}\rangle\langle\psi_{\text{in}}|$$

$$\eta = \exp\left(-L/L_{\text{att}}\right) = P_{\text{succ}} = \text{Tr}\left(\rho_{\text{out}}^{\text{PS}}\right)$$

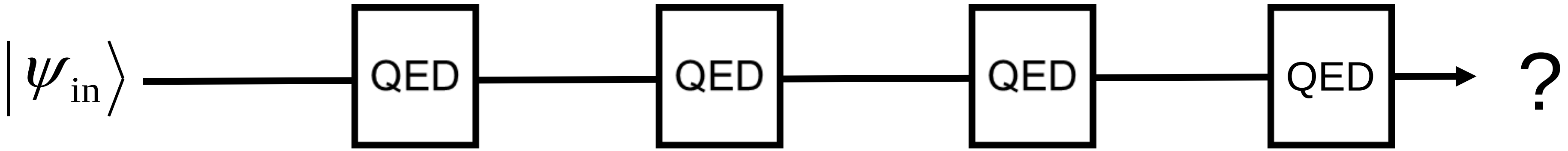
QED on Flying Qubits

$$|\psi_{\text{in}}\rangle = \alpha|10\rangle + \beta\,|01\rangle$$



QED on Flying Qubits

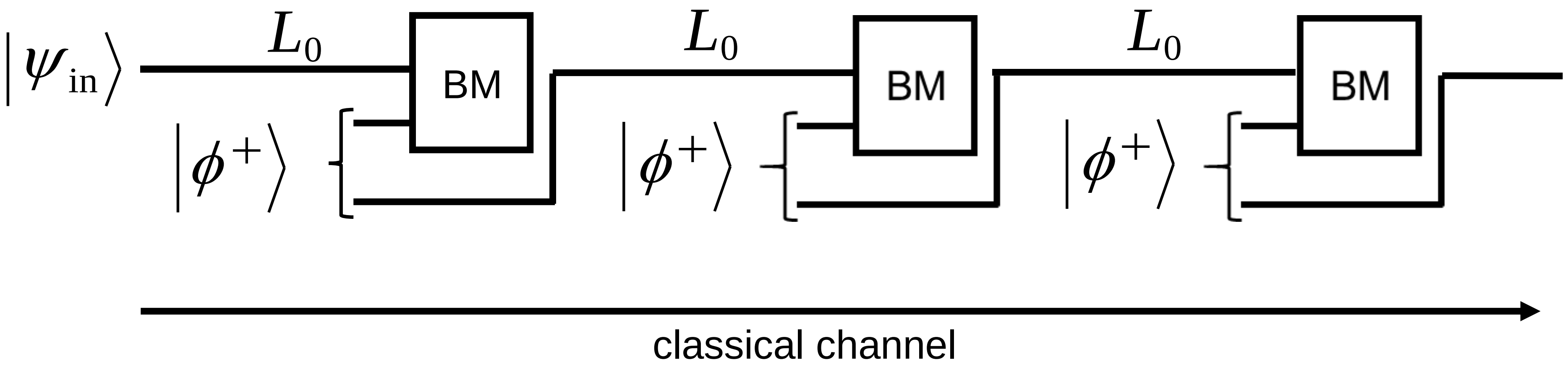
$$\left|\psi_{\text{in}}\right\rangle = \alpha\left|10\right\rangle + \beta\left|01\right\rangle$$



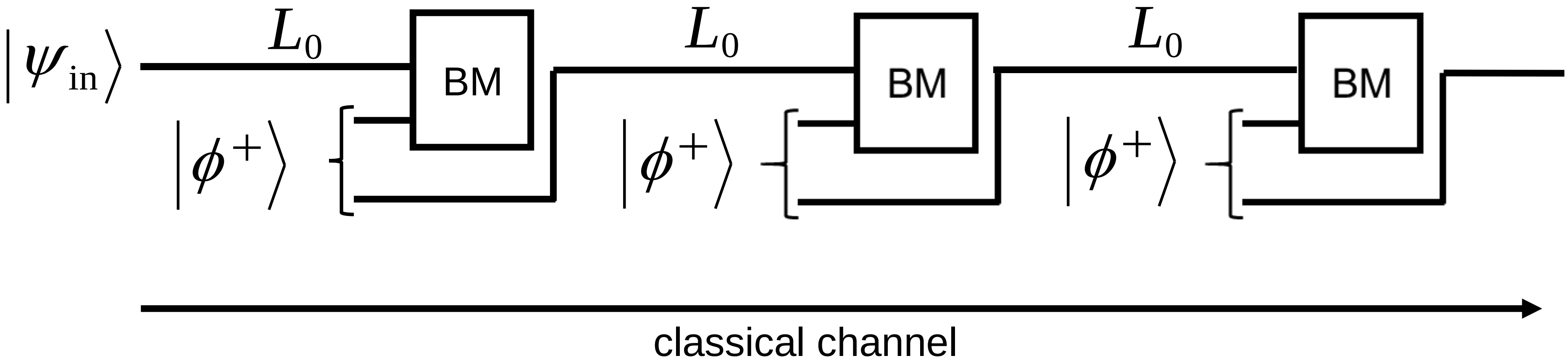
….need to detect the qubit non-destructively

QED on Flying Qubits

Bell measurement detects syndrome and „recovers“ in one step:
no loss = 2-photon detection,
photon lost =1-photon detection



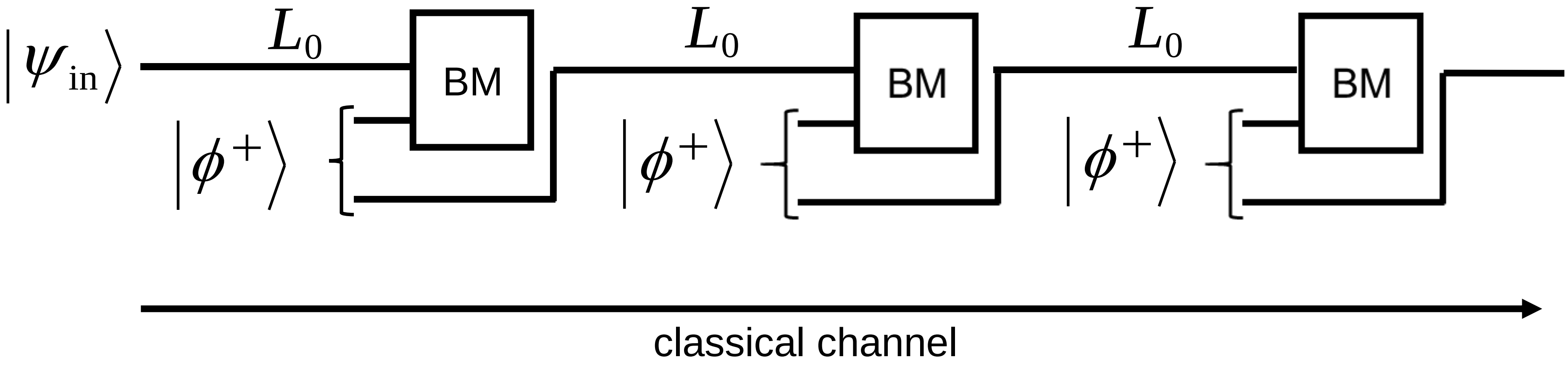
QED on Flying Qubits



Complications:

- ✓ on-demand generation of local Bell states
- ✓ Bell measurement with unit success probability
- ✓ never beats direct transmission

QED on Flying Qubits



$$P_{succ} = \left[P_{BM} \exp\left(-L_0 / L_{att}\right)\right]^{L/L_0} \leq \exp\left(-L / L_{att}\right)$$

(for any L_0)

Original Quantum Repeater

Essence of subexponential scaling:

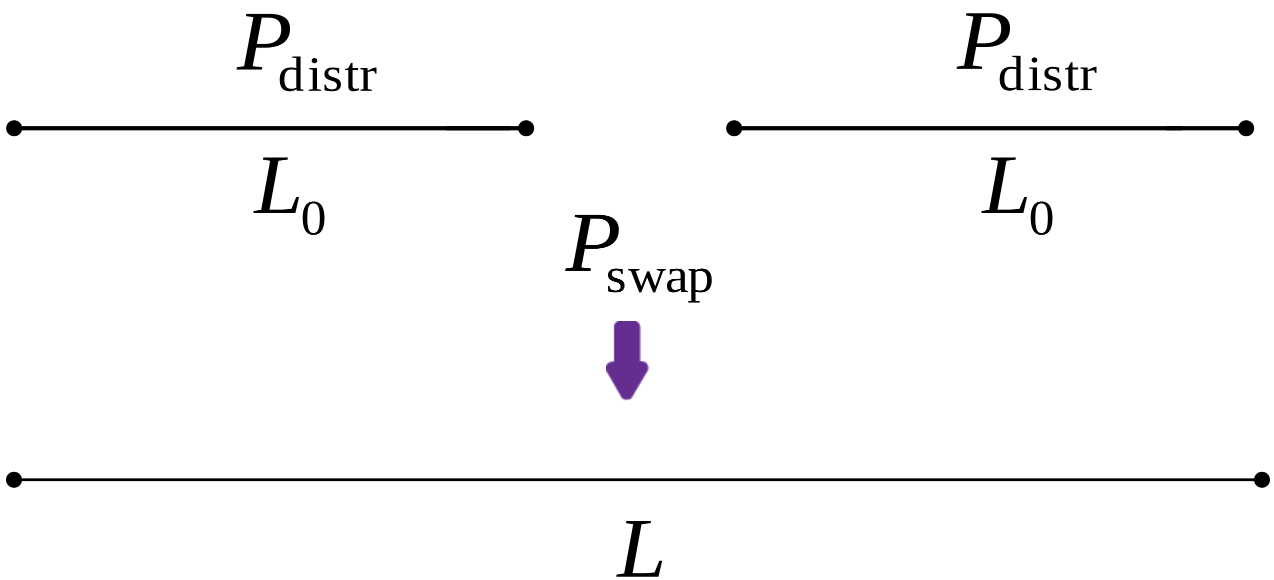
some form of quantum error detection

<u>and</u> quantum memories

Original Quantum Repeater

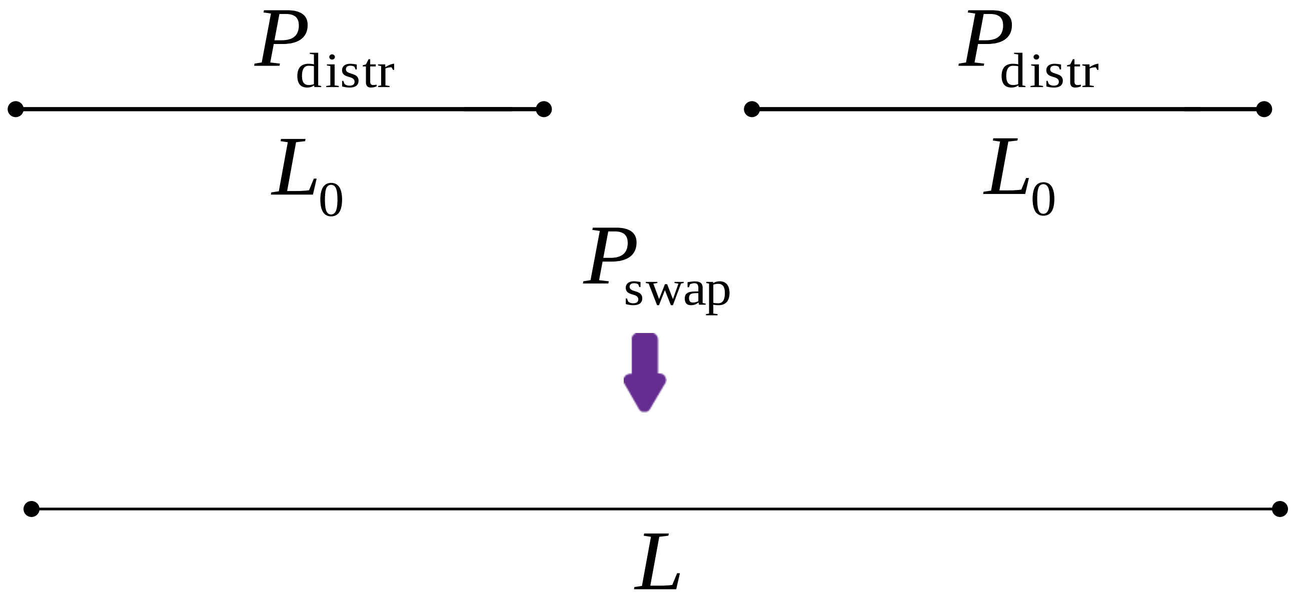
- ✓ distribute known, entangled states
- ✓ distribute different copies in each segment
- ✓ QED/entanglement purification
- ✓ quantum memories
- ✓ two-way classical communication

With Memories: Quantum Repeater



$$\text{Rate} \sim P_{\text{distr}} \left(\frac{2}{3} P_{\text{swap}} \right)^{\log_2 (L/L_0)} \sim \left(L/L_0 \right)^{\log_2 (2/3\, P_{\text{swap}})}$$

Without Memories: Quantum Relay

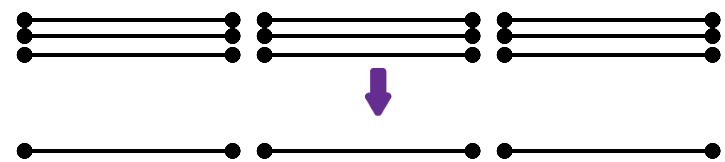


$$\text{Rate} \sim P_{\text{distr}}^{\,L/L_0} \cdot P_{\text{swap}}^{\,L/L_0 - 1}$$

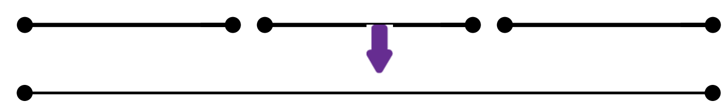
Original Quantum Repeater

✓ Entanglement Distribution

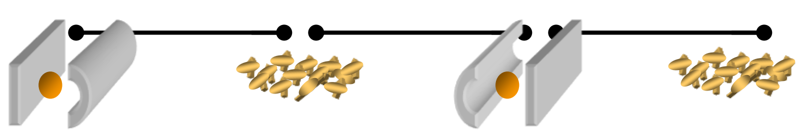
✓ Entanglement Purification
(Quantum Error Detection)



✓ Entanglement Swapping



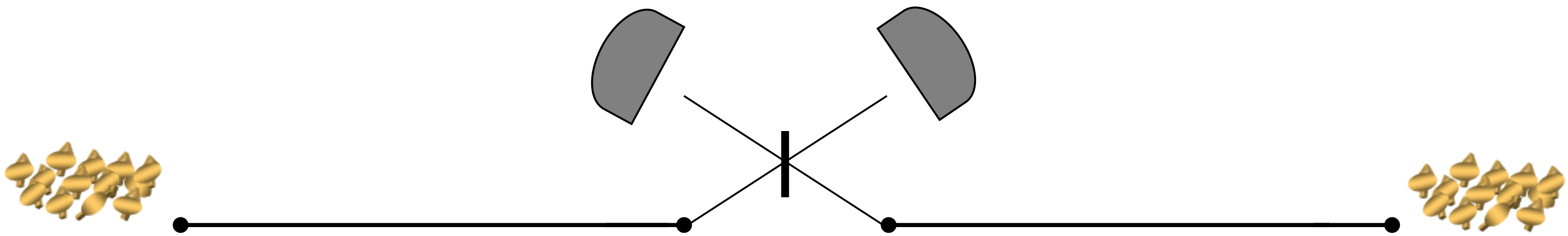
✓ Quantum Memories



H.J. Briegel *et al.*, PRL 81, 5932 (1998)

DLCZ Quantum Repeater

L.M. **D**uan, M.D. **L**ukin, J.I. **C**irac, P. **Z**oller, Nature **414**, 413 (2001)



$$I \otimes \$_\eta \to |00\rangle + r|11\rangle \qquad \$_\eta \otimes I \to |00\rangle + r|11\rangle$$

$$|01\rangle \pm |10\rangle$$

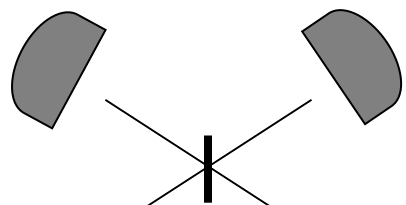
$$P_{\mathrm{distr}} \sim \eta \cdot r^2; \qquad F \approx 1 - O(r^2)$$

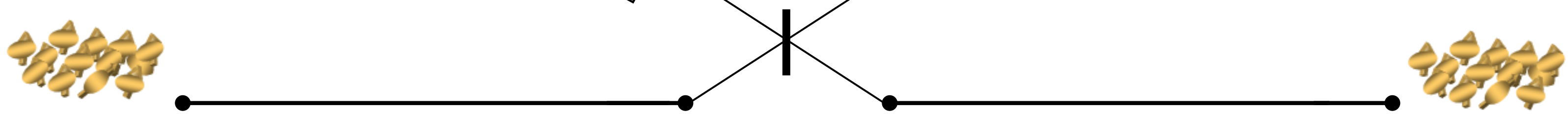
DLCZ Quantum Repeater

no-loss space

$$\cong |0000\rangle + r\sqrt{\eta}\left(|1100\rangle + |0011\rangle\right) + O(r^2)$$

$$r\sqrt{1-\eta} \rightarrow \begin{matrix} 1000 \\ 0001 \end{matrix}$$





$$I \otimes \$_\eta \rightarrow |00\rangle + r|11\rangle \qquad \$_\eta \otimes I \rightarrow |00\rangle + r|11\rangle$$

$$|01\rangle \pm |10\rangle$$

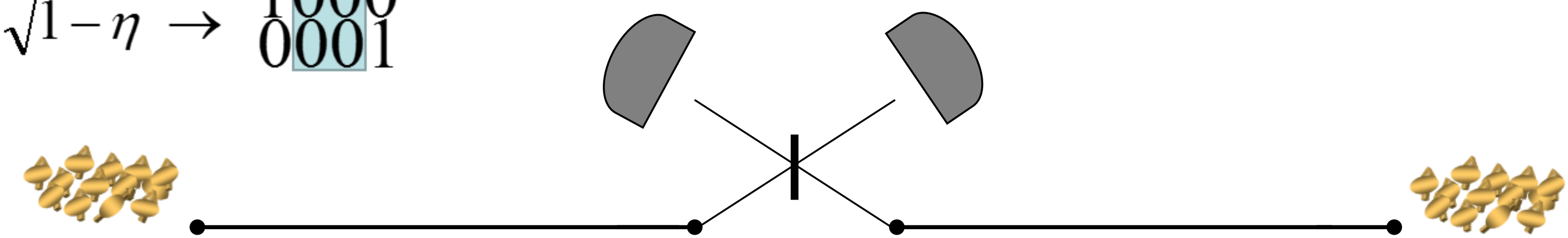
$$P_{\text{distr}} \sim \eta \cdot r^2; \qquad F \approx 1 - O(r^2)$$

DLCZ Quantum Repeater

loss space: only QED, not QEC!

$$\cong |0000\rangle + r\sqrt{\eta}\left(|1100\rangle + |0011\rangle\right) + O(r^2)$$

$r\sqrt{1-\eta} \rightarrow$ 1000, 0001



$I \otimes \$_\eta \rightarrow |00\rangle + r|11\rangle$ $\qquad$ $\$_\eta \otimes I \rightarrow |00\rangle + r|11\rangle$

$$|01\rangle \pm |10\rangle$$

$$P_{\mathrm{distr}} \sim \eta \cdot r^2; \qquad F \approx 1 - O(r^2)$$

Original Quantum Repeater

- ✓ distribute known, entangled states
- ✓ distribute different copies in each segment
- ✓ QED/entanglement purification
- ✓ quantum memories
- ✓ two-way classical communication

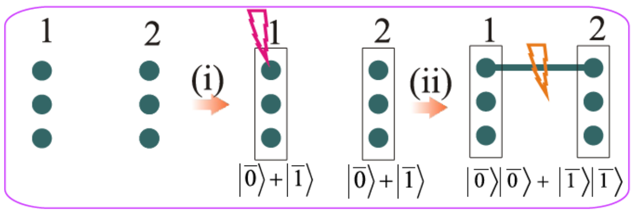
<u>Problems:</u> very slow, limited by CC rates, good memories required

$\text{Rate} \leq c / L$ e.g. 100Hz/1000km $\sim 1/\mathrm{O}(\mathrm{poly}(L))$

New Quantum Repeaters: Quantum Error Correction for Long-Distance Quantum Communication

Encoded Quantum Repeaters: Local Errors

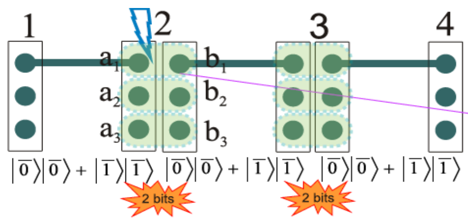
1. Encoded Generation

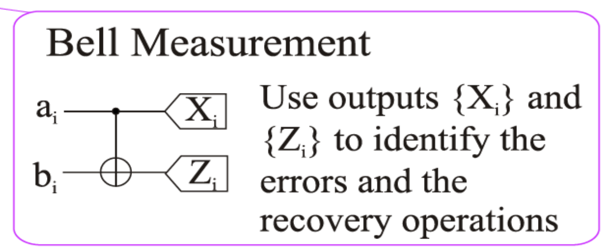


$$|\bar{0}\rangle = |000\rangle, \; |\bar{1}\rangle = |111\rangle$$

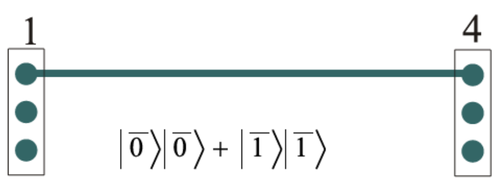
etc.

2. Encoded Connection





3. Pauli Frame



L. Jiang *et al.*, Phys. Rev. A **79**, 032325 (2009);
W.J. Munro *et al.*, Nat. Photon. **4,** 792 (2010)

$$\sim 1/O(\mathrm{poly}(\log(L)))$$

implementation-independent

S. Bratzik, H. Kampermann, and D. Bruß, PRA **89**, 032335 (2014) secret key rates in QKD

N.K. Bernardes and P.v.L., PRA **86,** 052301 (2012) HQR with encoding

Encoded Quantum Repeaters: Loss Errors

A.G. Fowler *et al.*, Phys. Rev. Lett. **104**, 180503 (2010) topological surface codes

W.J. Munro *et al.*, Nature Photon. **6**, 777 (2012) parity loss codes

K. Azuma, K. Tamaki, and H.-K. Lo, arXiv: 1309.7207 cluster states and feedforward

Photon Loss Codes

Leung‘s bosonic code:

$$|\bar{0}\rangle = \frac{|40\rangle + |04\rangle}{\sqrt{2}} \;,\; |\bar{1}\rangle = |22\rangle$$

exact

Leung‘s $[4,1]$ AD code:

$$|\bar{0}\rangle = \frac{|0000\rangle + |1111\rangle}{\sqrt{2}} \;,$$

approximate

$$|\bar{1}\rangle = \frac{|0011\rangle + |1100\rangle}{\sqrt{2}}$$

Photon Loss Codes

Quantum Parity Code (QPC):

$$|\pm\rangle^{(n,m)} = \frac{\left(|0\rangle^m \pm |1\rangle^m\right)^{\otimes n}}{\sqrt{2}^{\,n}}\ , \text{ with } |0\rangle^m = |10\rangle^{\otimes m}\ , |1\rangle^m = |01\rangle^{\otimes m}$$

$$|\bar{0}\rangle^{(n,m)} = \frac{\left(|+\rangle^{(n,m)} + |-\rangle^{(n,m)}\right)}{\sqrt{2}}\ , |\bar{1}\rangle^{(n,m)} = \frac{\left(|+\rangle^{(n,m)} - |-\rangle^{(n,m)}\right)}{\sqrt{2}}$$

QPC(n,n) corrects (n – 1) photon losses

T.C. Ralph, A.J.F. Hayes, and A. Gilchrist., PRL 95, 100501 (2005)

Photon Loss Codes

QPC(1,1):

$$\left|\pm\right\rangle^{(1,1)} = \frac{\left(\left|10\right\rangle \pm \left|01\right\rangle\right)}{\sqrt{2}}, \quad \left|\overline{0}\right\rangle^{(1,1)} = \left|10\right\rangle, \; \left|\overline{1}\right\rangle^{(1,1)} = \left|01\right\rangle$$

QPC(2,2):

$$\left|\overline{0}\right\rangle^{(2,2)} = \frac{\left|10101010\right\rangle + \left|01010101\right\rangle}{\sqrt{2}},$$

$$\left|\overline{1}\right\rangle^{(2,2)} = \frac{\left|10100101\right\rangle + \left|01011010\right\rangle}{\sqrt{2}}$$

$\Rightarrow$ $$C_{\mathrm{QPC}(2,2)} = C_{[4,1]} \circ C_{\mathrm{Dual-Rail}}$$ (is exact!)

Photon Loss Codes

Quantum Parity Code (QPC):

stabilizers for physical Pauli operators

$$Z_{ij}\,Z_{i,j+1} \quad i=1\ldots n\,,\;\; j=1\ldots(m-1)$$

$$\prod_{j=1}^{m} X_{ij} X_{i+1,j} \quad i=1\ldots(n-1)$$

$n(m-1)+n-1=nm-1$ independent stabilizers

QPC(2,2):

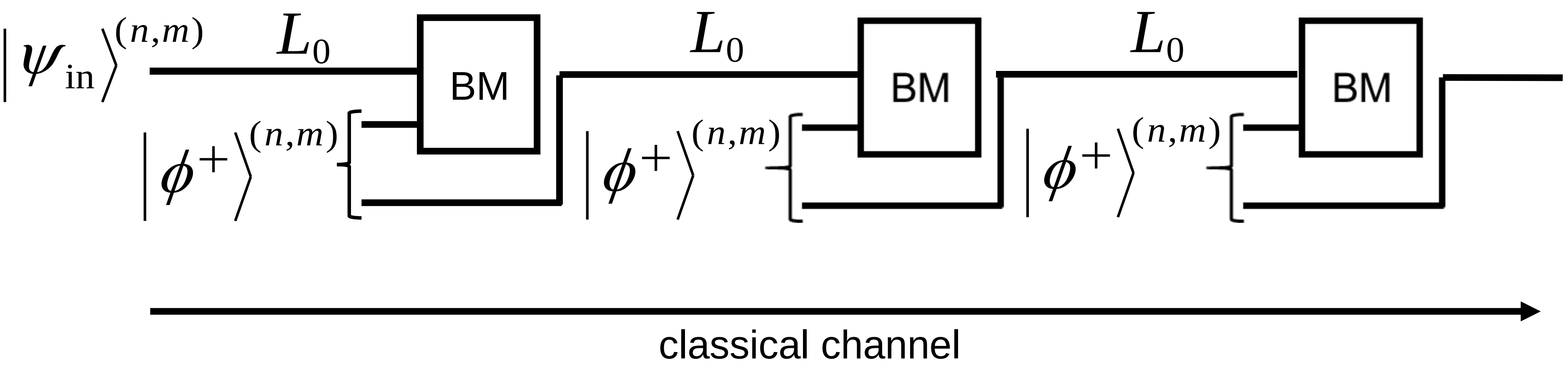
$$\langle Z_{11}Z_{12}, Z_{21}Z_{22}, X_{11}X_{21}X_{12}X_{22}\rangle = \langle ZZII, IIZZ, XXXX \rangle$$

like $[4,1]$ code

S. Muralidharan, J. Kim, N. Lütkenhaus, M.D. Lukin, and L. Jiang , PRL **112**, 250501 (2014)

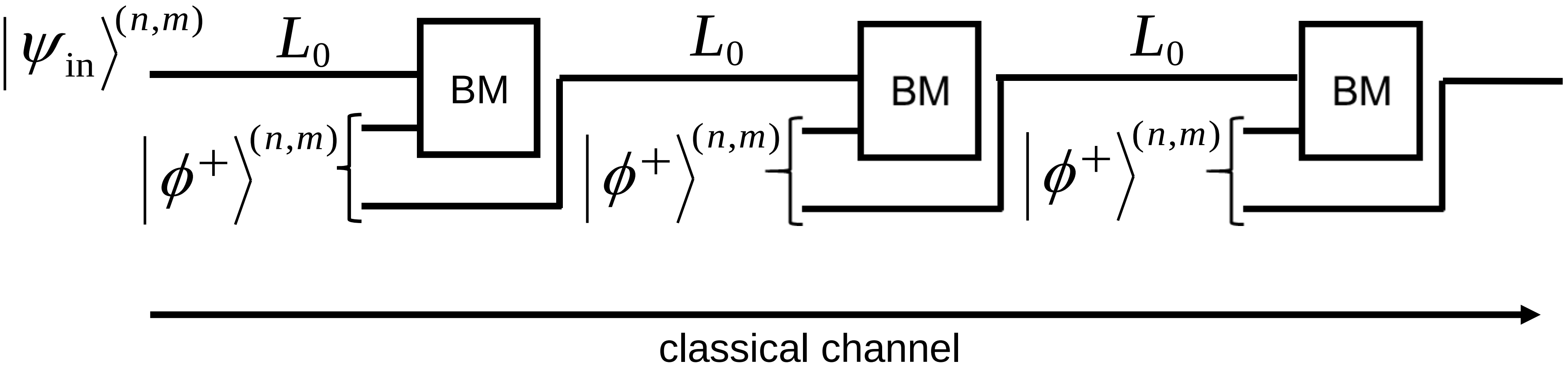
Ultrafast Quantum Communication

…replace DR-qubit/Bell states/BM's by QPC-encoded qubit/Bell states/BM's, use stabilizer formalism and exploit transversality of QPC code as a CSS code

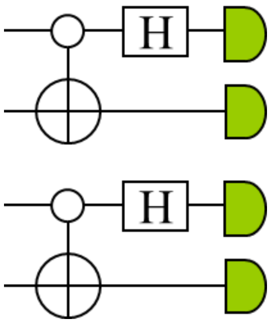


S. Muralidharan, J. Kim, N. Lütkenhaus, M.D. Lukin, and L. Jiang , PRL **112**, 250501 (2014)

Ultrafast Quantum Communication



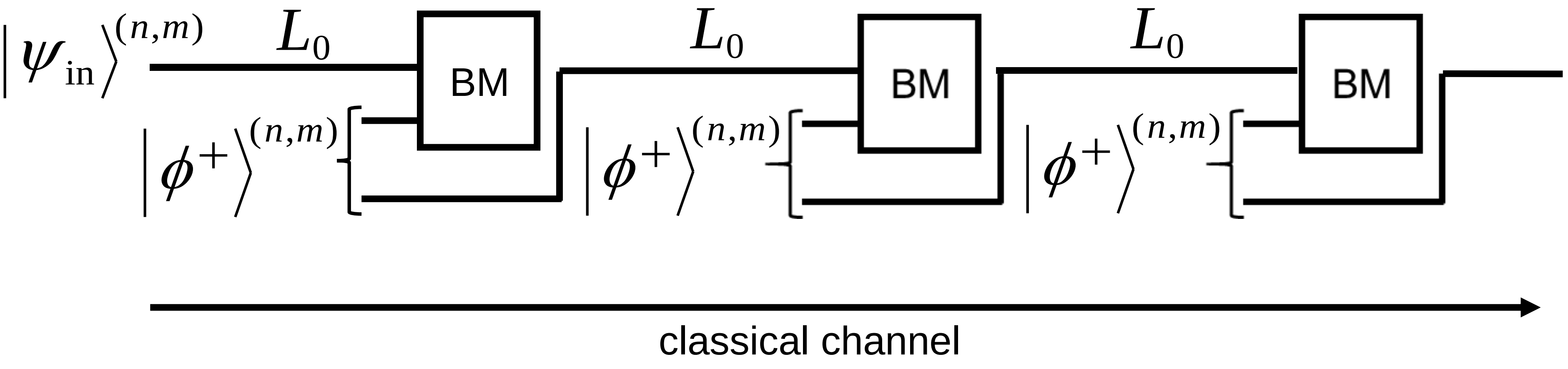
…many physical BM's for one logical BM
via many physical CNOTs and
many physical Hadamards:
need nonlinear operations, matter-light
interactions,…



Ultrafast Long-Distance Quantum Communication with Linear Optics

Linear-Optics Quantum Communication

…replace matter-qubit-based QPC-Bell states by optical QPC-Bell states and nonlinear light-matter interactions by static linear optics



Linear-Optics Quantum Communication

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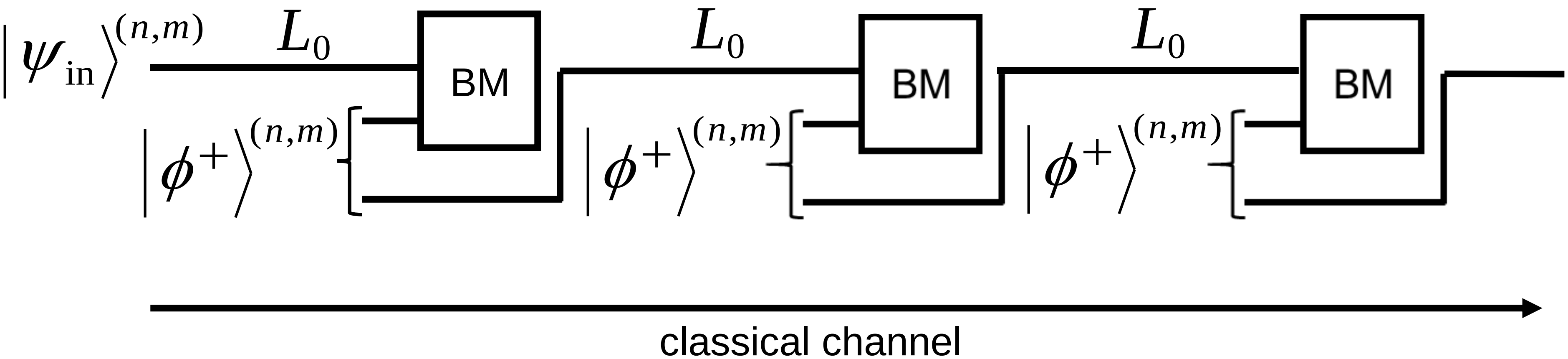
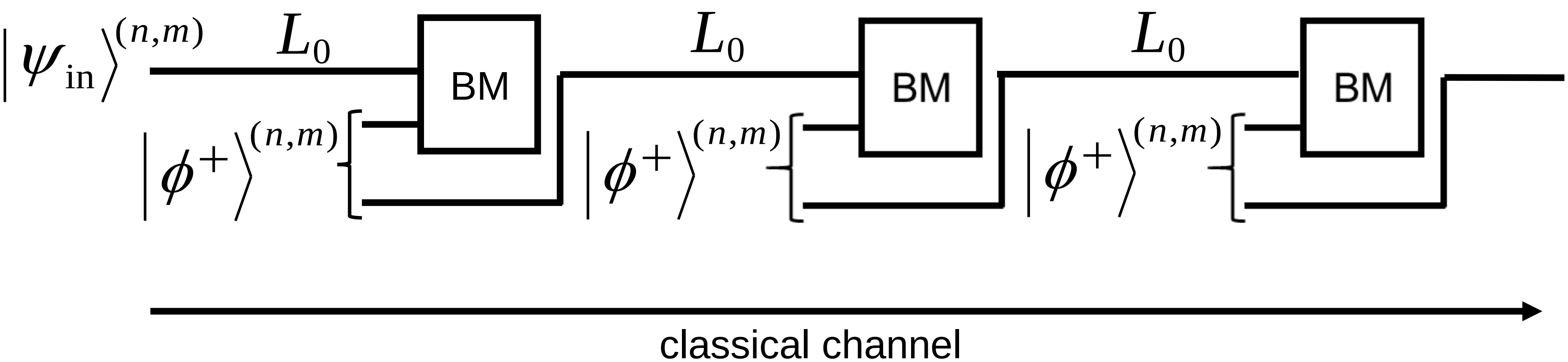


$$P_{\text{succ}} = \left[\sum_{l=0}^{nm} P_{\text{BM},l} \binom{nm}{l} \eta^{nm-l} (1-\eta)^l \right]^{L/L_0}$$

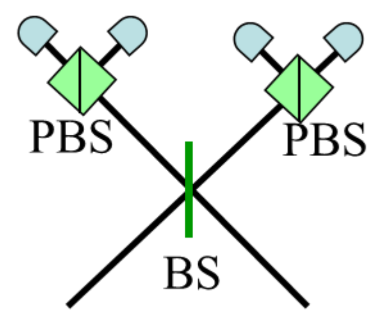
$$\eta = \exp\left(-L_0 / L_{\text{att}}\right)$$

Linear-Optics Quantum Communication

…replace matter-qubit-based QPC-Bell states by optical QPC-Bell states and nonlinear light-matter interactions by static linear optics



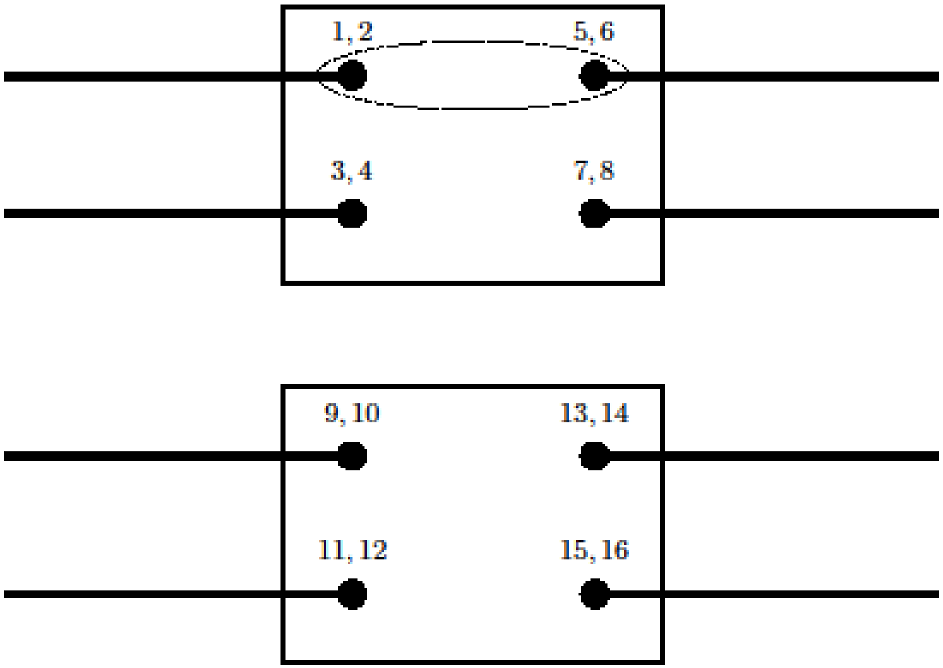
What is $P_{\text{BM},l}$?



Can we again exploit „transversality“?

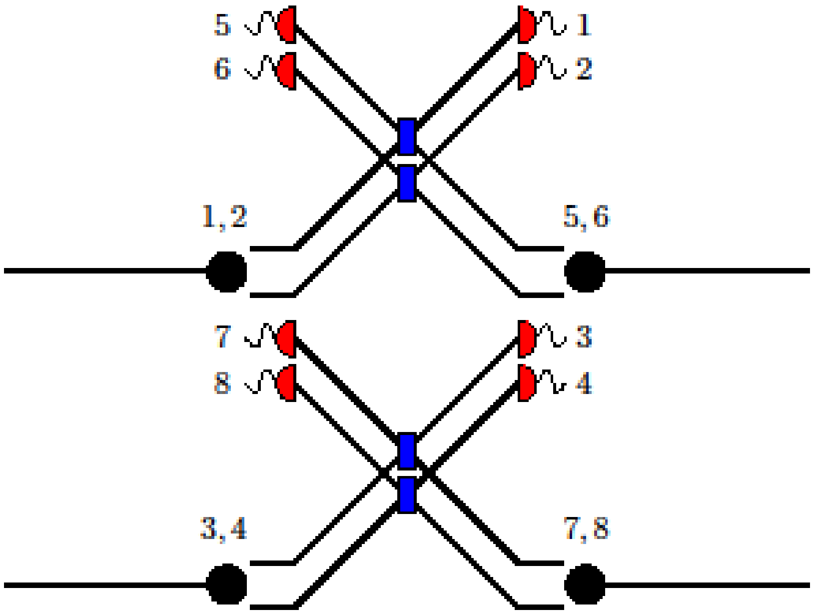
Linear-Optics Quantum Communication

BM of QPC(2,2) encoded Bell states:



Linear-Optics Quantum Communication

BM of QPC(2,2) encoded Bell states:



Linear-Optics Quantum Communication

(n,m)	0	1	2	3	4	5	6	7
(2,2)	75	50						
(3,2)	87.5	75	40					
(4,3)	93.75	87.5	77.27	61.36	40.91	20.45	5.84	
(5,3)	96.88	93.75	88.39	79.12	65.11	47.20	28.32	12.59
(10,5)	99.90	99.80	99.63	99.31	98.79	97.96	96.72	94.94

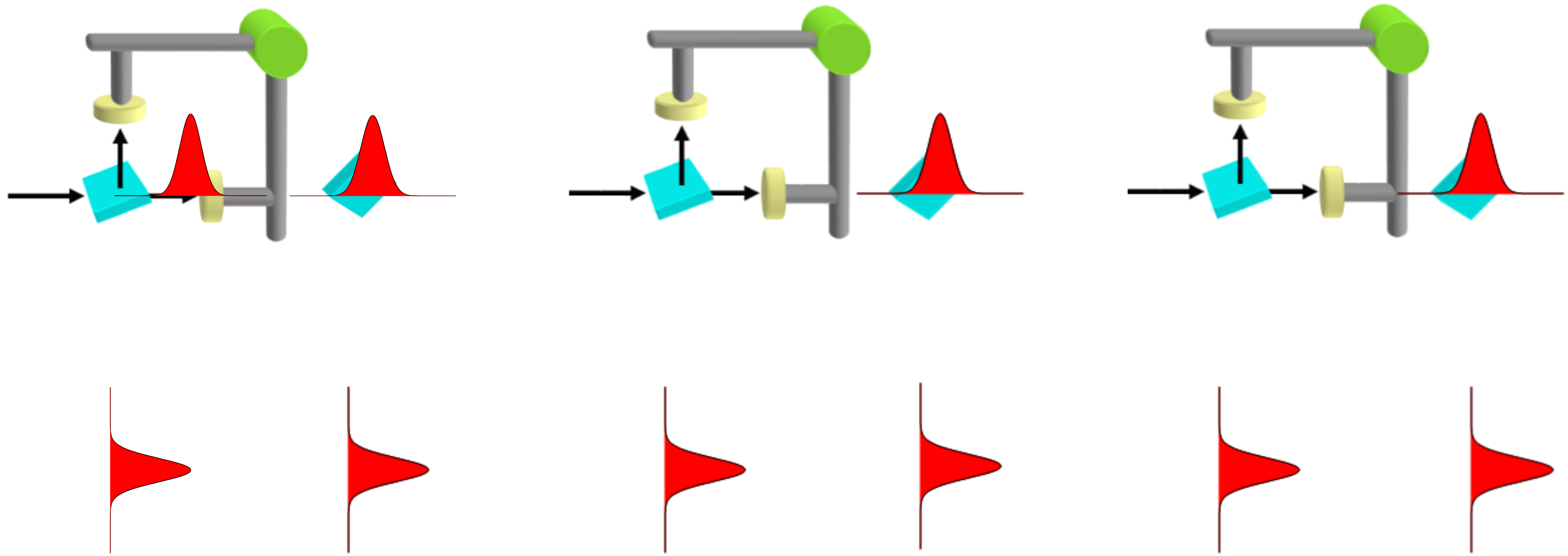
(n,m)	0	1	2	3	4	5	6	7
(2,2)	75	50	7.14					
(3,2)	87.5	75	43.18	10.91	1.21			
(4,3)	93.75	87.5	77.72	63.44	46.22	29.35	15.91	7.21
(5,3)	96.88	93.75	88.58	80.08	67.99	53.30	38.03	24.38
(10,5)	99.90	99.80	99.63	99.32	98.82	98.05	96.91	95.31

Table 1: Success probabilities p_l in % of the one-sided (top) and the symmetric BM (bottom) given the total number of photons lost l.

$$P_{\mathrm{BM},l=0} = 1 - 2^{-n}$$

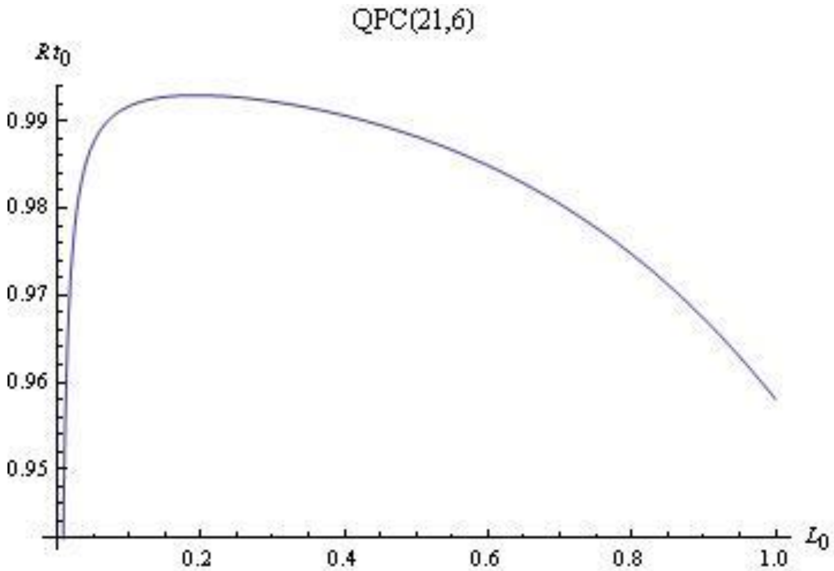
QPC-encoded BM works asymptotically well with linear optics (no loss) and it even still works in the presence of losses !

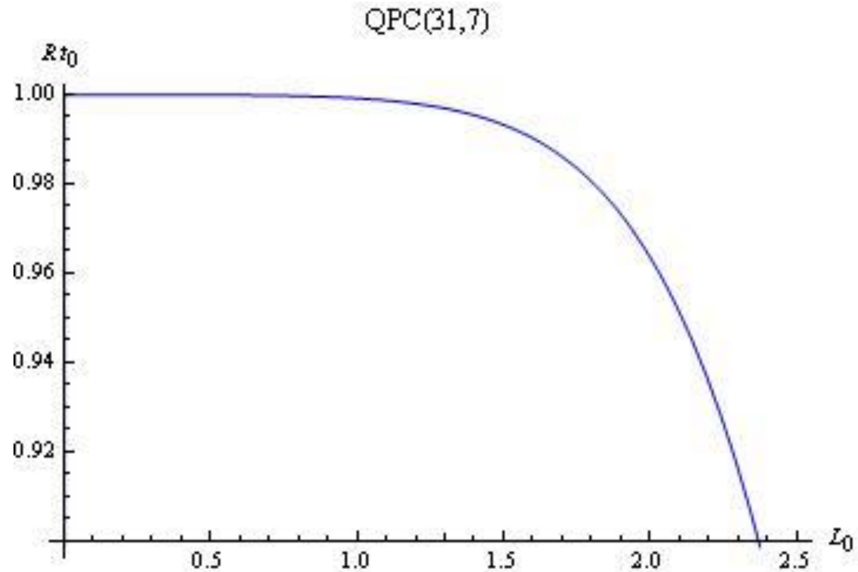
Linear-Optics Quantum Communication



Success Probabilities (Temporal Cost)

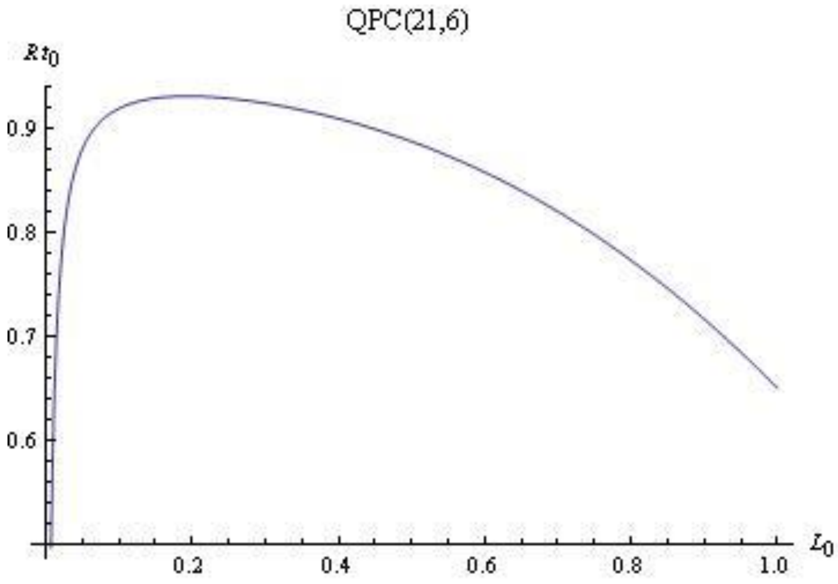
Rate $R = P_{\text{succ}} / t_0$

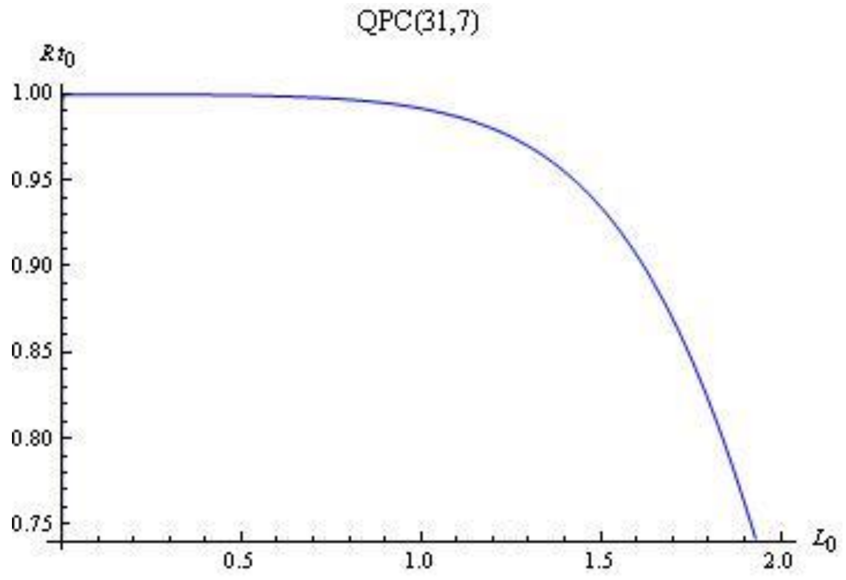




1000 km

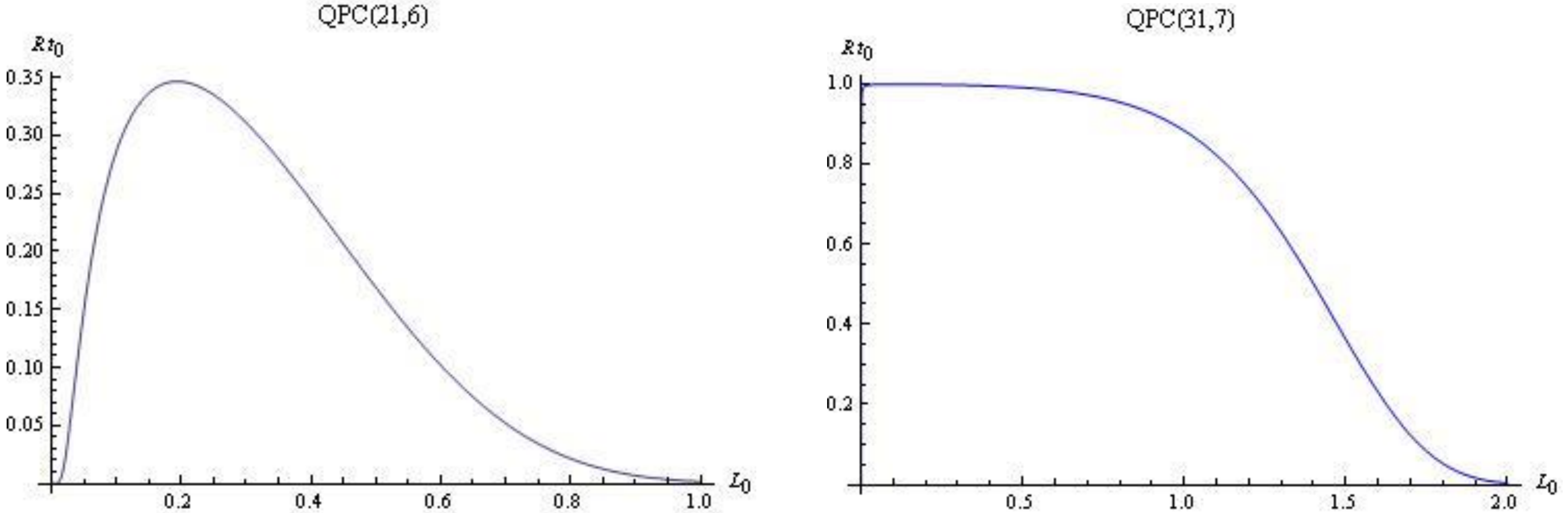
Success Probabilities (Temporal Cost)





10000 km

Success Probabilities (Temporal Cost)

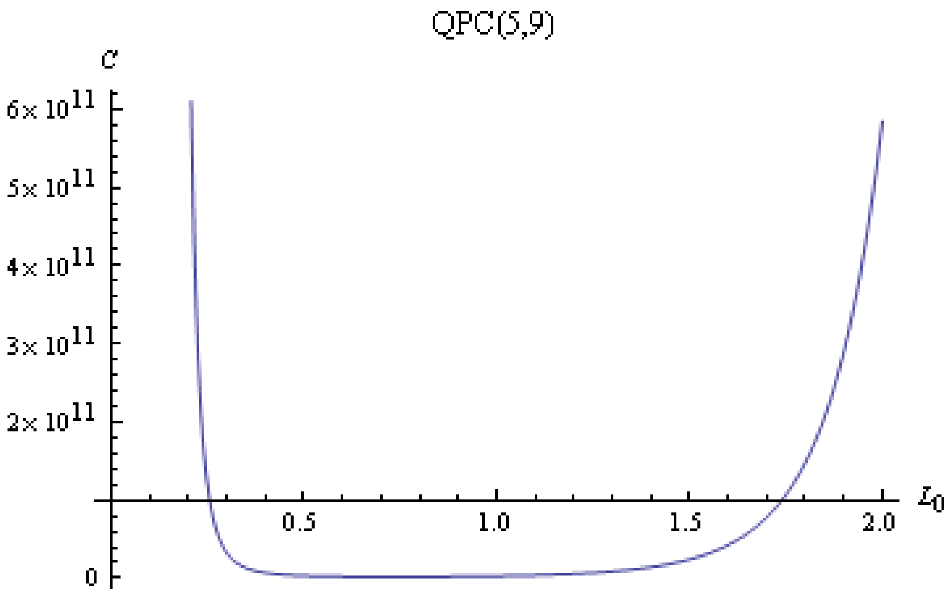


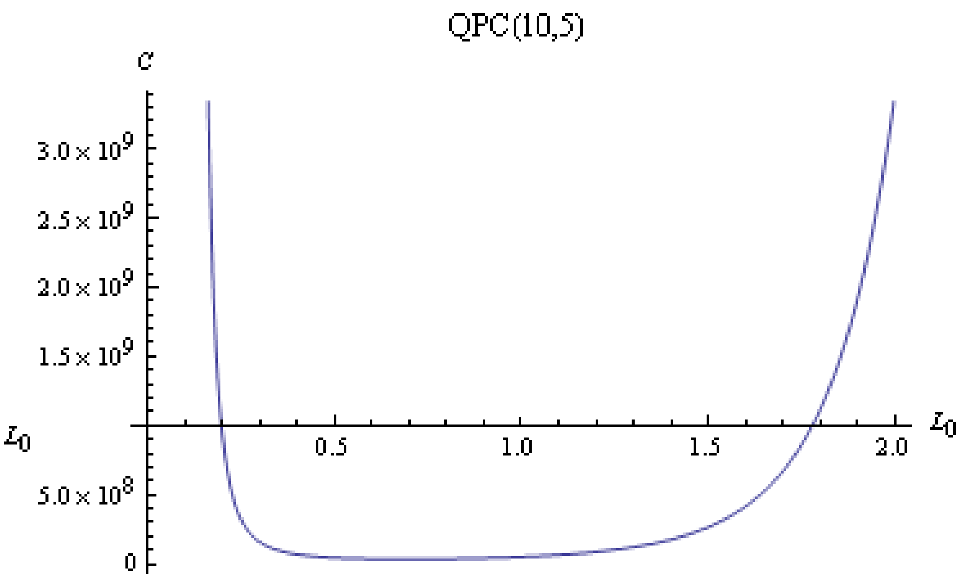
150000 km

Total Cost

$$C = (2nm/R)L/L_0 = (2nmt_0/P_{succ})L/L_0$$

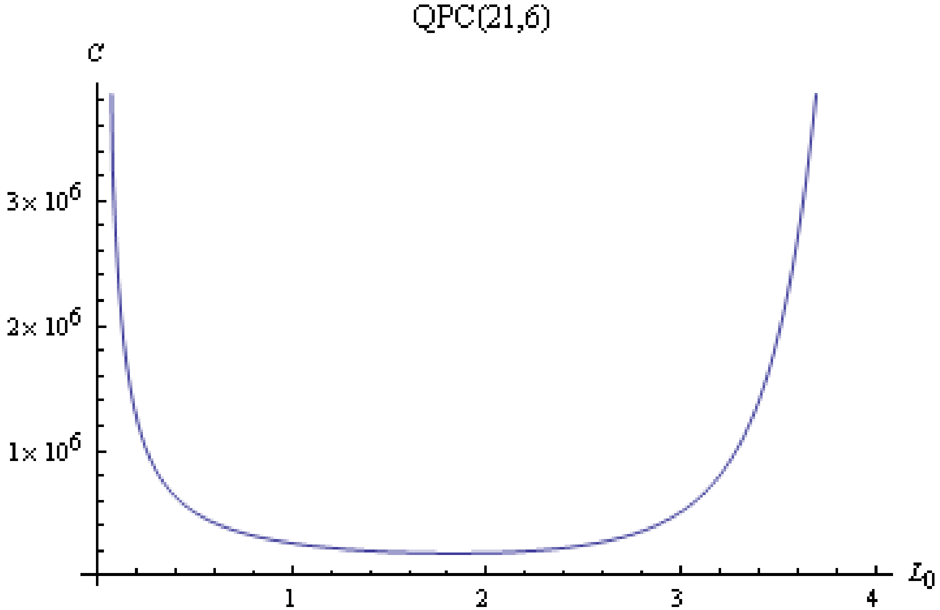
S. Muralidharan, J. Kim, N. Lütkenhaus, M.D. Lukin, and L. Jiang , PRL **112**, 250501 (2014)

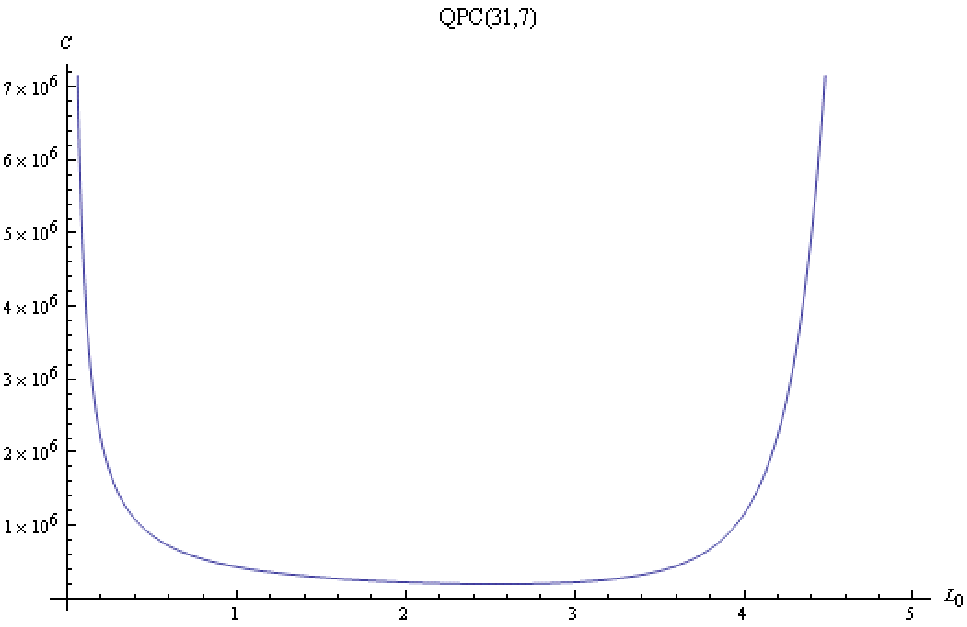




1000 km

Total Cost





1000 km

Summary

- ✓ Standard quantum repeaters using QED are scalable in principle, but slow
- ✓ New generation of quantum repeaters using QEC significantly improve the rates
- ✓ Ultrafast loss-code-based scheme is implementable with linear optics

QR rates near CC rates, only limited by local times