XS-Stabilizer

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QEC14

http://arxiv.org/abs/1404.5327

Definition

- Pauli-S group: $\mathcal{P}_n^s = \langle \alpha, X, S \rangle^{\otimes n}$

$$\alpha = \sqrt{i} \quad S = \mathrm{diag}(1, i) \quad S^{-1} X S = -iXZ$$

- Given $G = \langle g_1, \ldots, g_m \rangle \subset \mathcal{P}_n^s$

We call a state $|\psi\rangle$ XS-stabilizer state if (uniquely)

$$g_j |\psi\rangle = |\psi\rangle$$

When not unique, we call it XS-stabilizer code

Outline

- Operator picture
- State picture

Operator picture

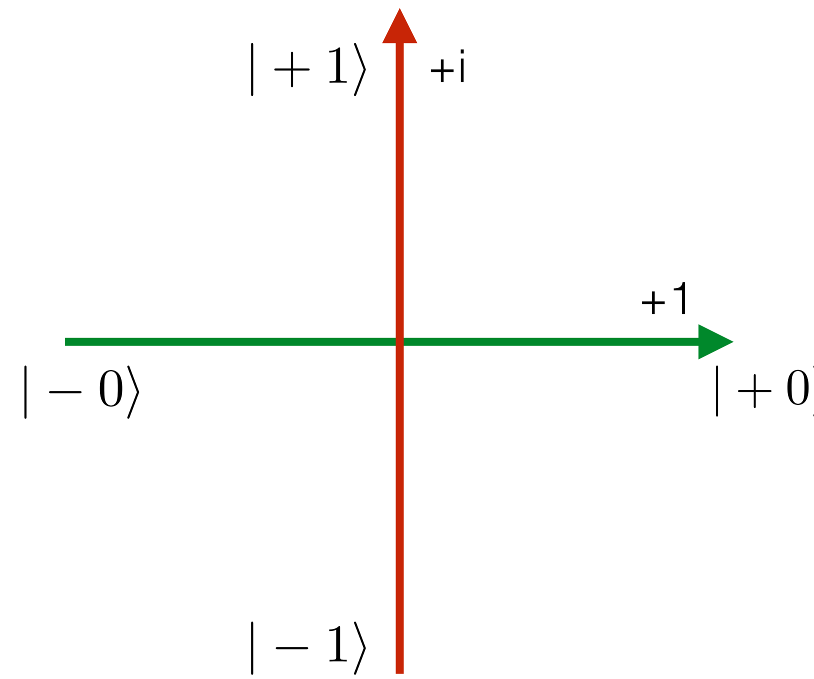
Starting point of Pauli stabilizer

- Either commute or anti-commute
- Each generator evenly split the Hilbert space
- Commutativity allows consecutively splitting

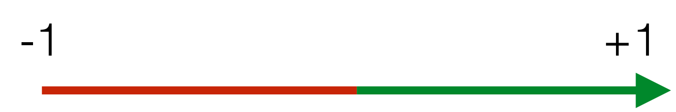
Example

$P(X \otimes S)$ is Hermitian

$X \otimes S$



$$P \longleftarrow (X \otimes S)^2 = I \otimes Z$$

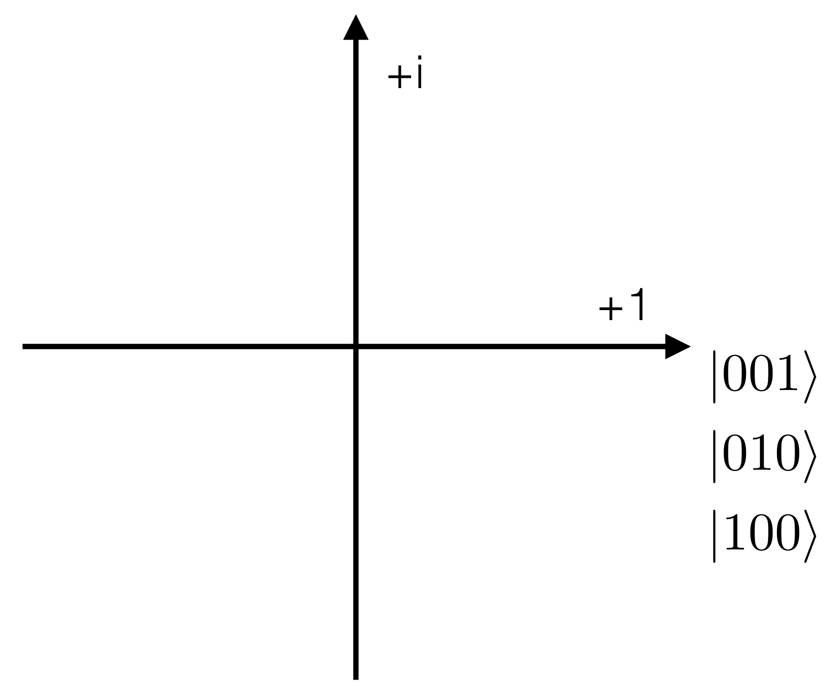


Example

$$i^3 S \otimes S \otimes S$$

Only one of x_1, x_2, x_3 is equal to 1

(Positive) 1-in-3 SAT problem
NP-Complete



Two operators

1. Commute and independent
2. Commute but not fully independent

$$g_1 = X \otimes S \otimes I$$
$$g_2 = I \otimes S \otimes X$$
$$g_1^2 = I \otimes Z \otimes I = g_2^2$$

Two operators

3. Partially commute

$$g_1 = X \otimes X \otimes S \otimes S \longrightarrow P_1$$

$$g_2 = S \otimes S \otimes X \otimes X \longrightarrow P_2$$

$$g_1 g_2 g_1^{-1} g_2^{-1} = Z \otimes Z \otimes Z \otimes Z$$

$$\downarrow$$

$$P_{12} = \frac{1}{2}(1 + Z \otimes Z \otimes Z \otimes Z)$$

Commuting projectors

$$g_1|\psi\rangle = g_2|\psi\rangle = |\psi\rangle$$

$$\Updownarrow$$

$$P_1 P_{12}|\psi\rangle = P_2 P_{12}|\psi\rangle = P_{12}|\psi\rangle = |\psi\rangle$$

Find codeword state

- Given $G = \langle g_1, \ldots, g_m \rangle \subset \mathcal{P}_n^s$, define diagonal subgroup as G_D.
- We can construct a codeword state $|\psi_x\rangle$, if we can find a computational basis $|x\rangle$ stabilized by G_D.
- When G_D is generated by Z-type operators, this procedure is efficient.

Van den Nest 2011

Diagonal subgroup

Each element of G has the form: $\mathcal{Z} g_1^{x_1} \ldots g_m^{x_m}$,

where $\mathcal{Z}$ is generated by $\{g_j^2\} \cup \{g_j g_k g_j^{-1} g_k^{-1}\}$

So we can write down a set of generators of G_D by using linear algebra

Operator picture

- Properties of operators
- Computational complexity
- Equivalent commuting projectors
- Find code states

The state picture

The state picture

- Concrete
- Easiest way to utilize the uniqueness condition
- (Innsbruck-Munich influence)

Example

$$g_1 = X \otimes S^3 \otimes S^3 \otimes S \otimes X \otimes X,$$
$$g_2 = S^3 \otimes X \otimes S^3 \otimes X \otimes S \otimes X,$$
$$g_3 = S^3 \otimes S^3 \otimes X \otimes X \otimes X \otimes S.$$

$$\sum_{x_j=0}^{1} (-1)^{x_1 x_2 x_3} |x_1, x_2, x_3, x_2 \oplus x_3, x_1 \oplus x_3, x_1 \oplus x_2\rangle$$

Mechanism

$$Z \bigotimes Z \bigotimes Z$$

$$\sum_{x_1, x_2} |x_1, x_2, x_1 \oplus x_2\rangle$$

Mechanism

$$X \otimes Z$$

$$|0, x_2\rangle \leftrightarrow (-1)^{x_2}|1, x_2\rangle$$

$$\downarrow$$

$$\sum (-1)^{x_1 x_2} |x_1, x_2\rangle$$

Mechanism

$$X \otimes S \otimes \cdots$$

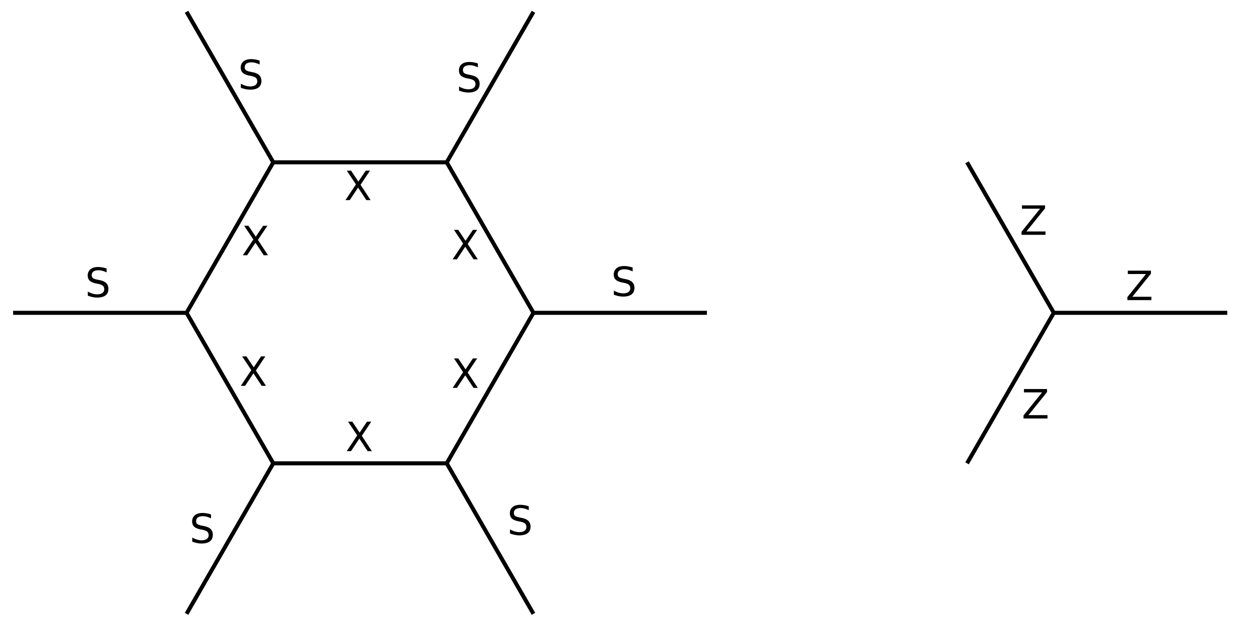
$$\boxed{|0, x_2 \oplus x_3, \cdots\rangle \leftrightarrow i^{x_2+x_3}(-1)^{x_2 x_3}|1, x_2 \oplus x_3, \cdots\rangle}$$

$$\sum i^{x_1(x_2+x_3)}(-1)^{x_1 x_2 x_3}|x_1, x_2 \oplus x_3, \cdots\rangle$$

Bravyi, Haah 2012

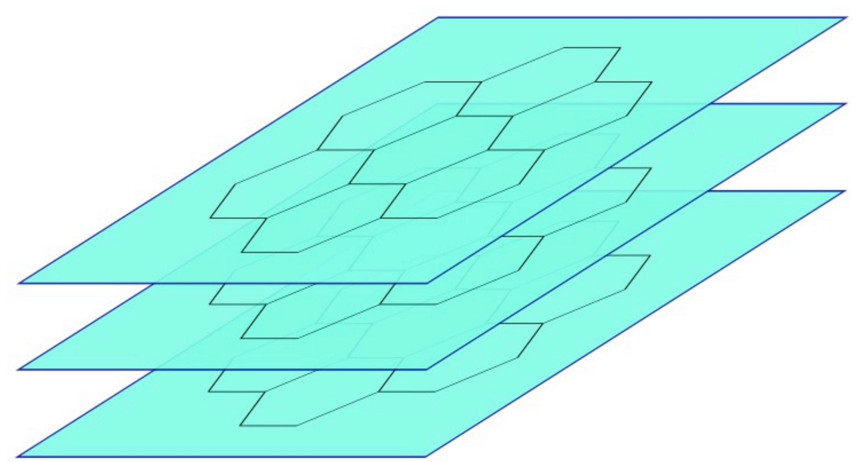
Twisted quantum double

- Double semion: $\displaystyle\sum_{x \text{ is close loops}} (-1)^{\text{number of loops}} |x\rangle$



Twisted quantum double

- twisted double on $\mathbb{Z}_2^n$



Flip a (plaquette) loop,
add a quadratic phase

Hu, Wan, Wu 2012

Mechanism

$$X \otimes S \otimes \cdots$$

$$\boxed{|0, x_2 \oplus x_3, \cdots\rangle \leftrightarrow i^{x_2+x_3}(-1)^{x_2 x_3}|1, x_2 \oplus x_3, \cdots\rangle}$$

$$\sum i^{x_1(x_2+x_3)}(-1)^{x_1 x_2 x_3}|x_1, x_2 \oplus x_3, \cdots\rangle$$

S-CZ gadget

$$\sum_{x_1,x_2} |x_1, x_2, x_1 \oplus x_2\rangle$$

$$S^{-1} \otimes S^{-1} \otimes S = CZ_{12} \otimes I$$

Why quadratic?

$$\text{If we have } X \otimes \sqrt{S} \otimes \cdots$$

$$(X \otimes \sqrt{S} \otimes \cdots)^2 = I \otimes S \otimes \cdots$$

Hard to make it compatible with the string intuition

Discussion

- Should we add CZ to the Pauli-S group?
- There's some tradeoff. Choose what is the most convenient for you

Other funny facts

- XS states have very similar entanglement properties compared to Pauli states (~Flammia, Hamma, Hughes, Wen)
- Double semion (and probably other twisted double model) have transversal logical-X gate

Some error deserves not to be corrected

Thanks