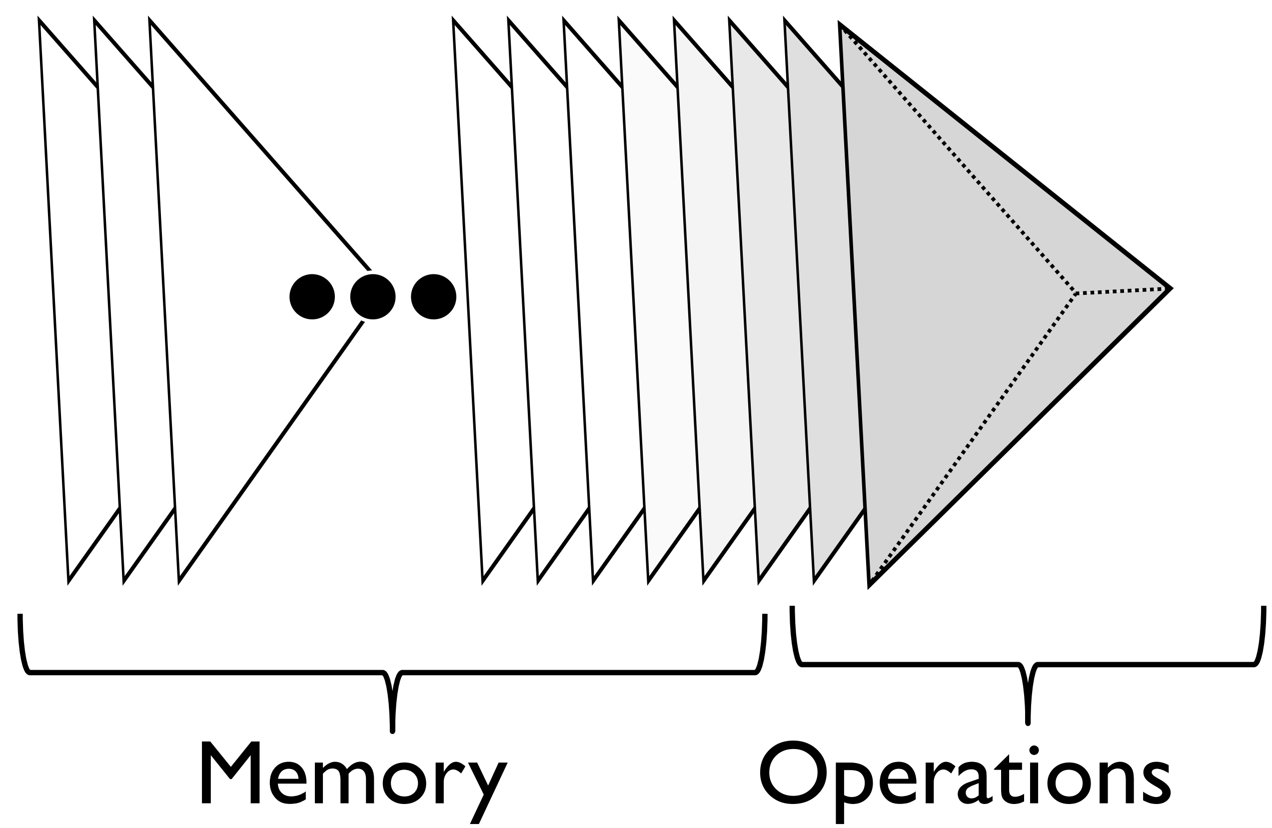
color codes
(are fun)

Héctor Bombín

summary

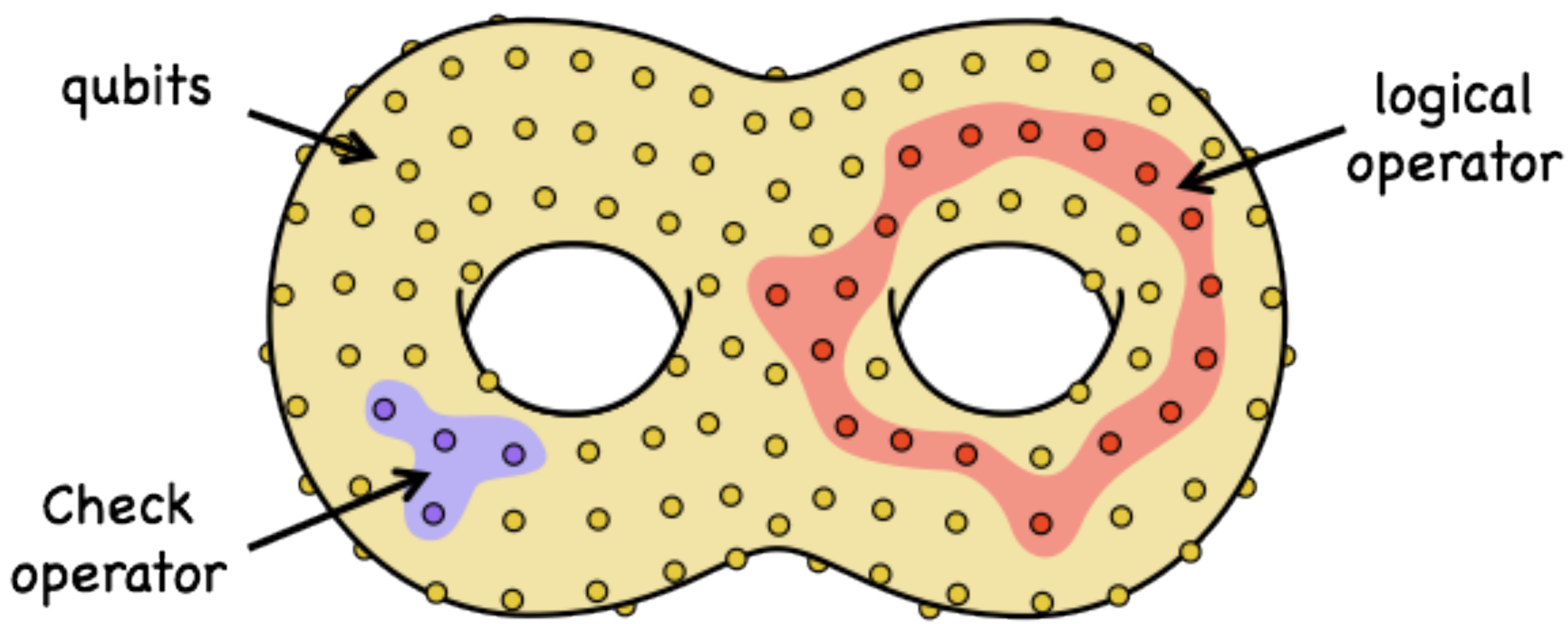
- Fault-tolerant QC in **3D** qubit lattice
- **6-local** quantum ops + global classical computation
- **Constant** time overhead (disregarding *efficient* CC)



ingredients

topological stabilizer codes

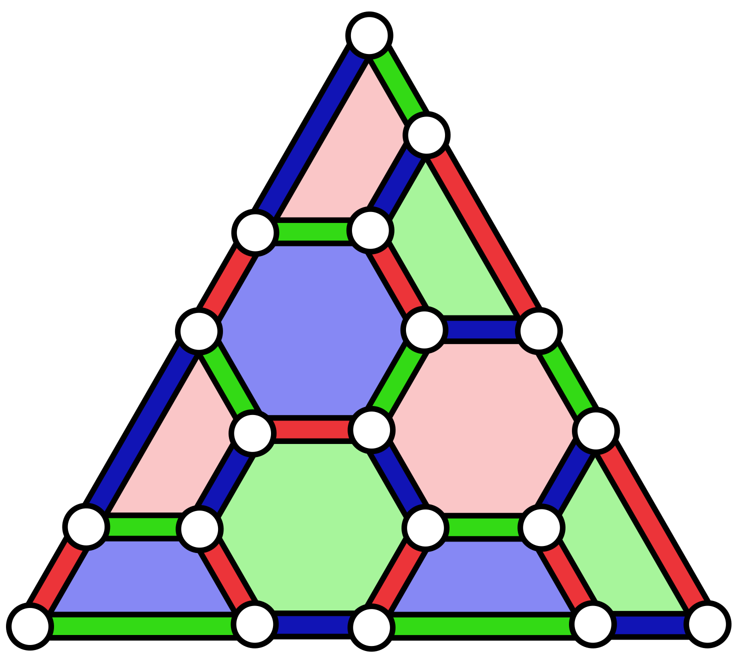
Kitaev '97



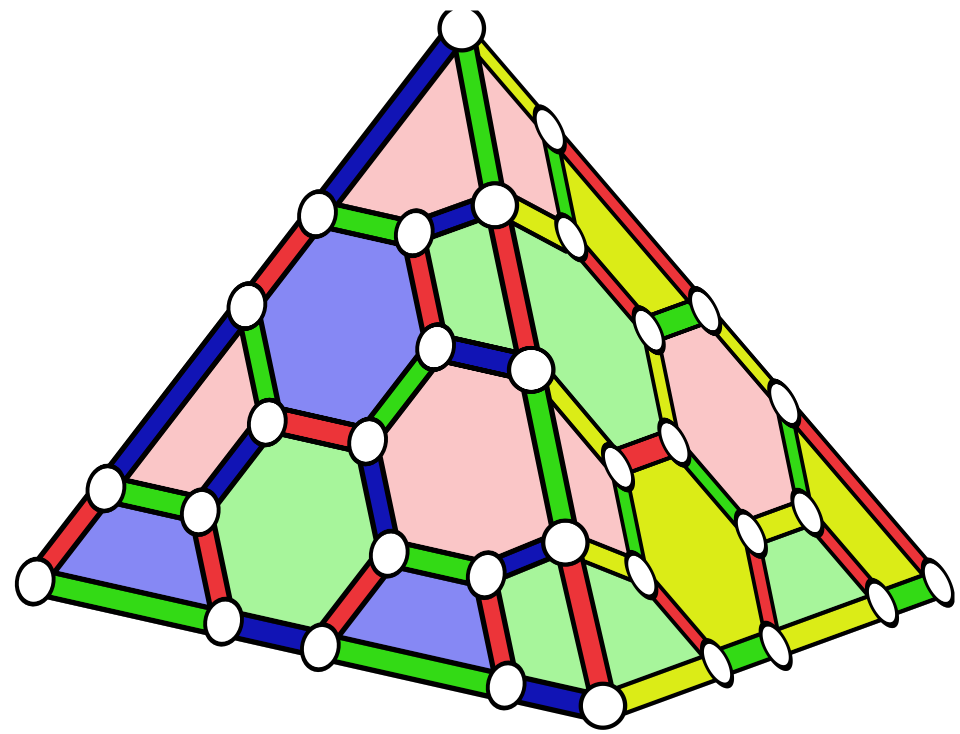
Geometrically local codes

color codes

Topology + transversality



Clifford group



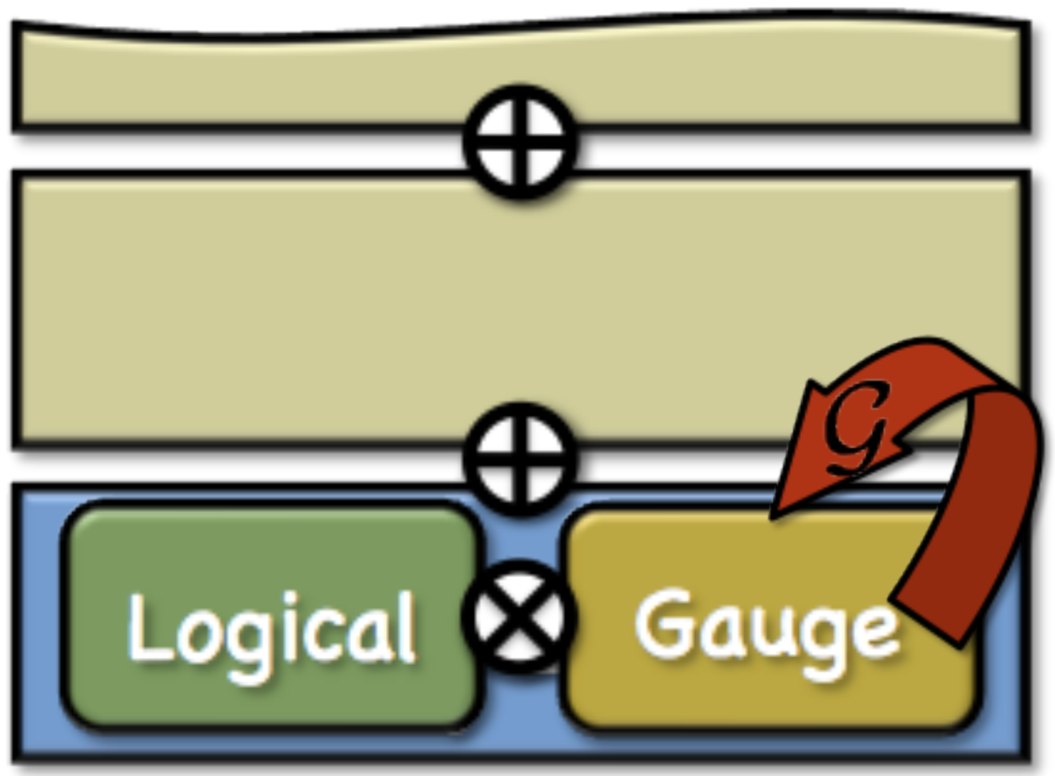
CNOT + T

- Optimal for every *D* (Bravyi-König constraint)

Bombín '13

subsystem stabilizer codes

Poulin '05



Gauge group

$$\mathcal{G} \subseteq \mathcal{P}$$

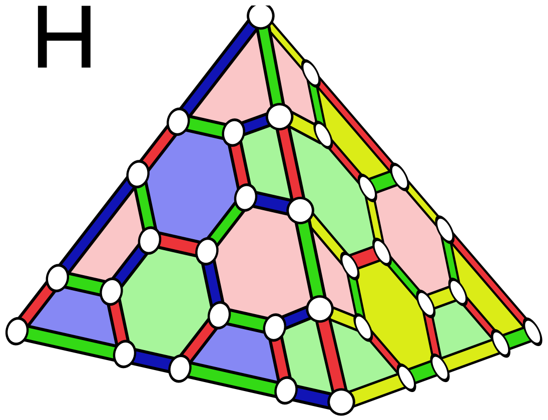
$$\mathcal{S} \propto \mathrm{Center}(\mathcal{G})$$

Logical operators: bare $\dfrac{\mathcal{Z}(\mathcal{G})}{\mathcal{S}}$ dressed $\dfrac{\mathcal{Z}(\mathcal{S})}{\mathcal{G}}$

subsystem stabilizer codes

Poulin '05

- Gauge degrees of freedom can be local in TSC Bombín '10
  - more localized measurements (even 2-local)
- 3D gauge color codes:
  - 6-local measurements
  - self-dual (beyond homology!), transversal H



gauge fixing

Paetznick & Reichardt ‘13

- Combine properties of different codes! (universality…)
- Switch between different codes with
  - shared set of representative bare logical ops
  - either (equivalently)

$$\mathcal{S}_1 \subseteq \mathcal{S}_2 \qquad \text{or} \qquad \mathcal{G}_2 \subseteq \mathcal{G}_1$$

1 $\longrightarrow$ 2 fix the value of some gauge generators

1 $\longleftarrow$ 2 do nothing

code splitting

- gauge fixing can split a code if applied to
  - collection of codes defined on disjoint sets of n_i qubits
  - single code with $\sum n_i$ qubits

$$\mathcal{L} = \prod_i \mathcal{L}_i, \qquad \mathcal{G}_i \subseteq \mathcal{G}.$$

- Color codes: **dimensional jump!**

single-shot error correction

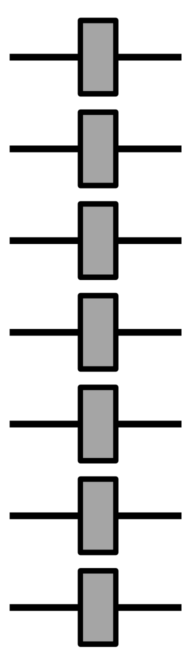
Bombín ‘14

- *Problem*: measurement errors in error detection
- *Solution*: repeat measurements as much as needed Shor ‘96
- *Alternative*: single-shot fault-tolerant error correction
  - robust error correction strategy (redundancy)
  - single round of local measurements
  - local measurement errors produce local residual errors
- Useful in gauge-fixing, initialization…

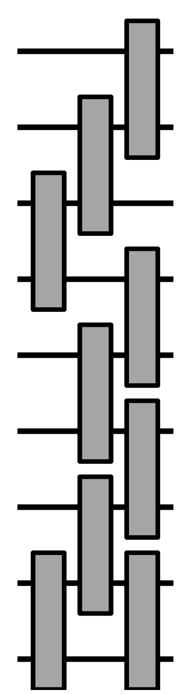
single-shot error correction

locality

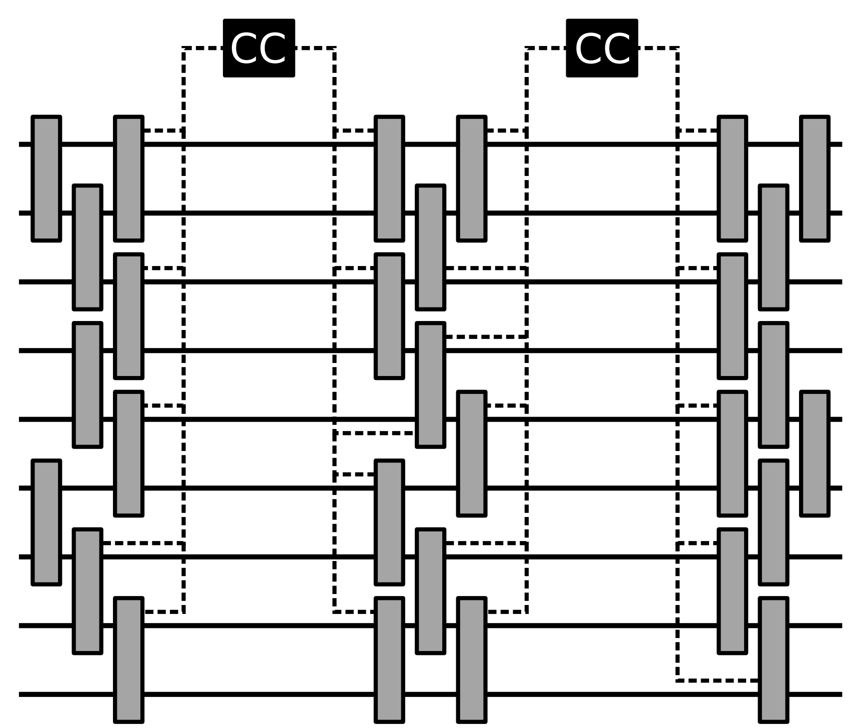
- Locality of errors is crucial for QEC
- Operations should preserve locality



transversal



local



quantum-local ☠

quantum-local error correction

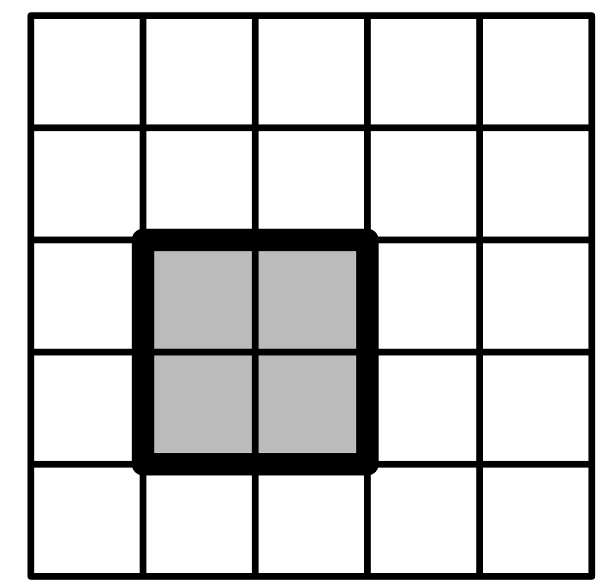
- Local stabilizer codes = q-local *ideal* error correction
- Global classical computation needed to decode
- If measurements are noisy, correction might introduce large errors
- If local measurement errors only give local residual errors: q-local *fault-tolerant* error correction
- This is single-shot (fault-tolerant quantum) error correction

self-correction

- TSCs give examples of topological order:
  - gapped local quantum Hamiltonian, topological degeneracy of ground state
  - GS = code, excitations = syndromes
- Some phases survive at finite T: self-correction
- There exists a connection between self-correction and single-shot error correction: **confinement**

Ising model

- Simplest (classical) self-correcting system
- Critical temperature T_C if $D>1$
- Below T_C excitations are confined: energy dominates over entropy
- Stable bit for exponentially long time on system size

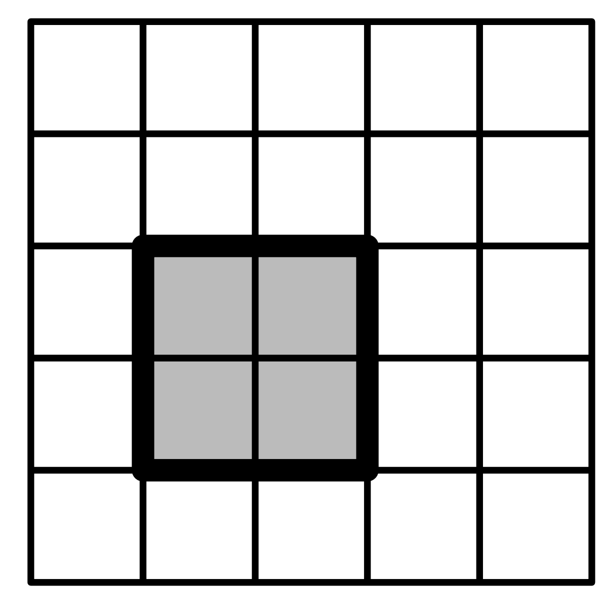


repetition code *à la* Ising

- stabilizer code for bit-flip errors
- qubits = faces, stabilizers = edges

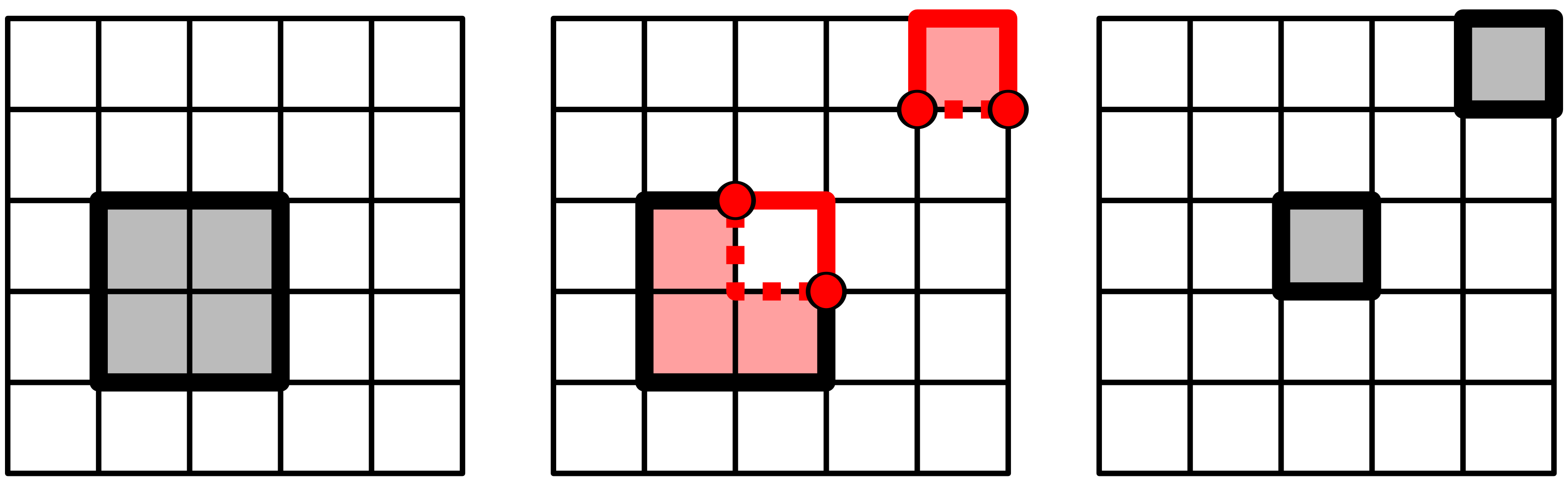
$$Z_e := Z_i Z_j$$

- syndrome composed of loops
- for low local noise, confined loops



noisy error correction

- assume noise in measurements only, not at correction stage
- goal: residual loops should be confined



before | error correction | after

- effective wrong measurements give residual syndrome

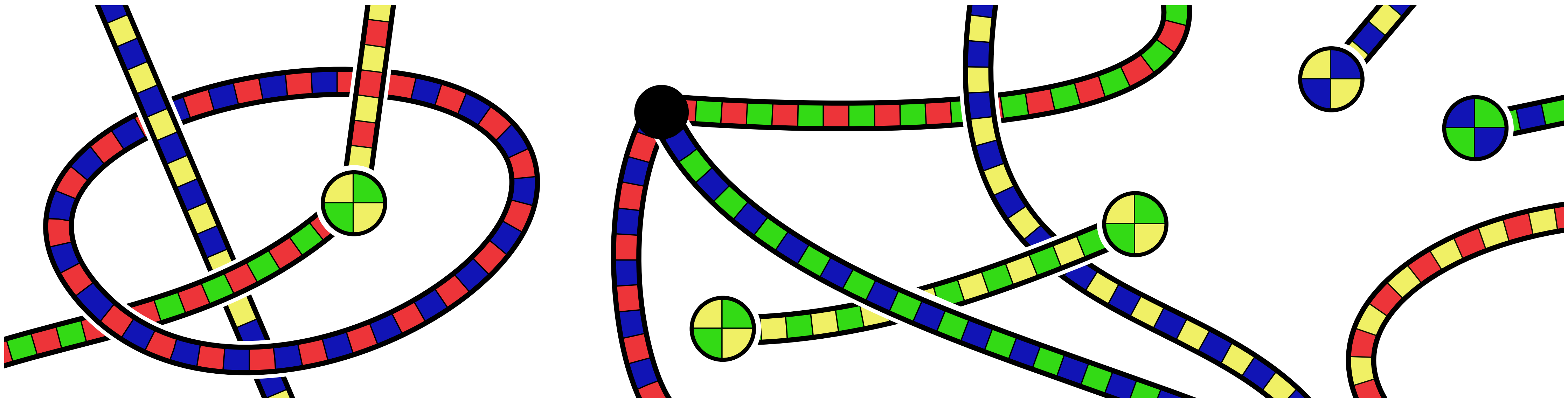
spatial dimension

- 1D Ising model: no confinement of punctual excitations
- 1D repetition code: no confinement of syndrome under measurement errors
- Confinement mechanism: excitations are extended objects
- Full quantum self-correction seems to require D>3

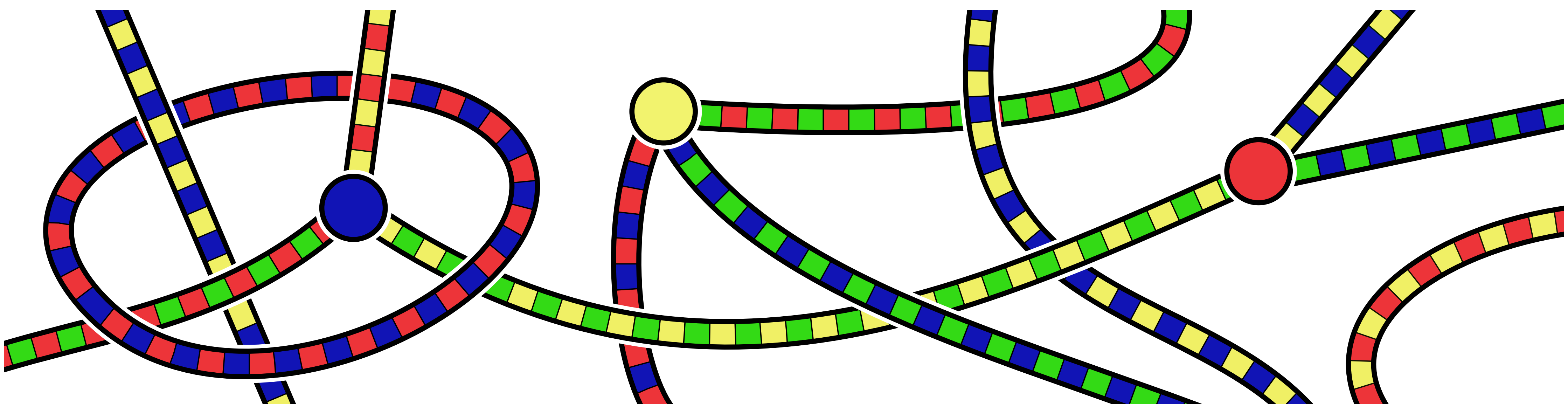
confinement in 3D

- 3D gauge color codes:
  - errors: string-like
  - syndrome: endpoints
- Direct measurement of syndrome: no confinement
- Instead, obtain it from **gauge syndrome**: string-net
- Another application of gauge structure

confinement in 3D



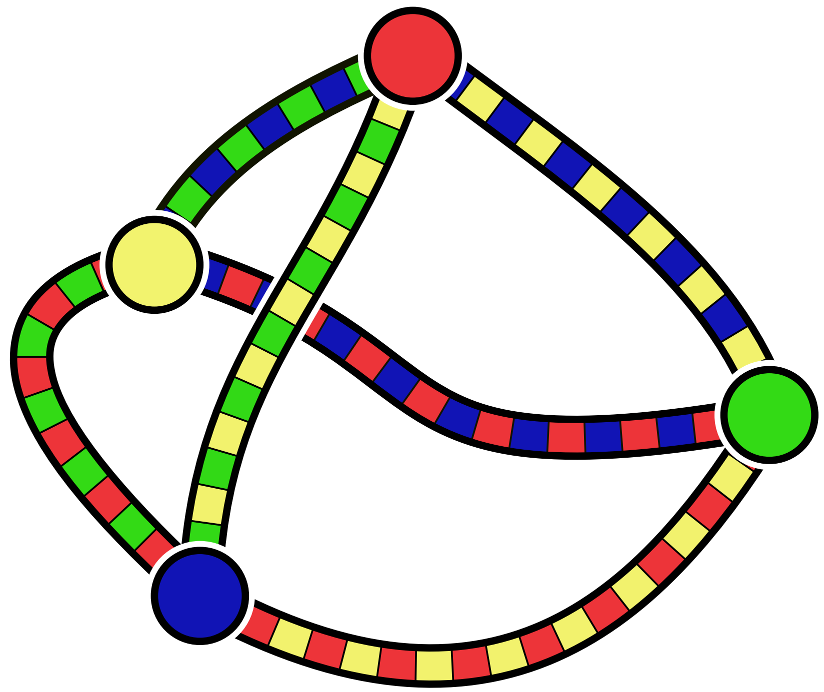
faulty gauge syndrome: endpoints = syndrome of faults



repaired gauge syndrome: branching points = syndrome

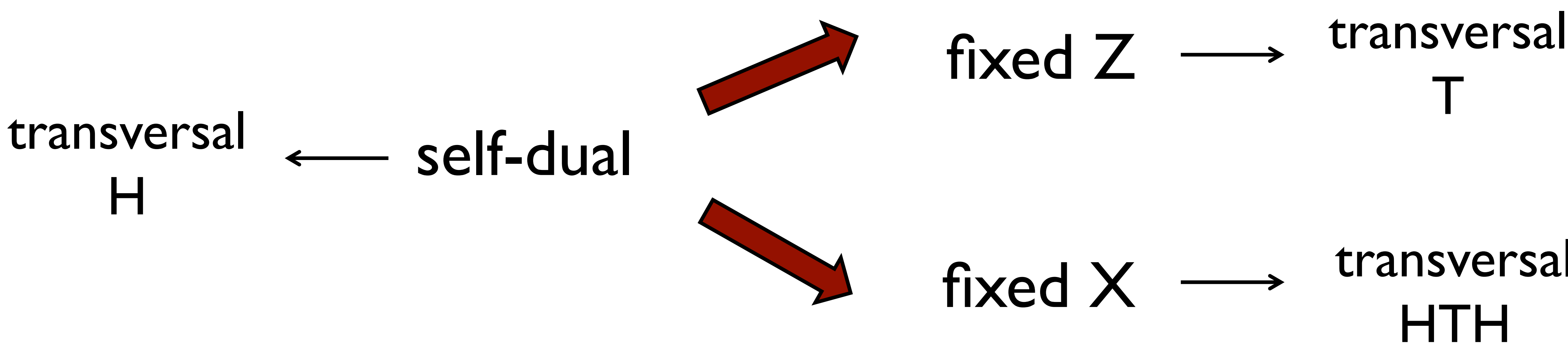
confinement in 3D

- The gauge syndrome is **not** confined, it is random except for the fixed branching points
- The (effective) wrong part of the gauge syndrome **is** confined
- Each connected component has branching points with neutral charge (*i.e.* locally correctable).
- Branching points exhibit **charge confinement**



confinement in 3D

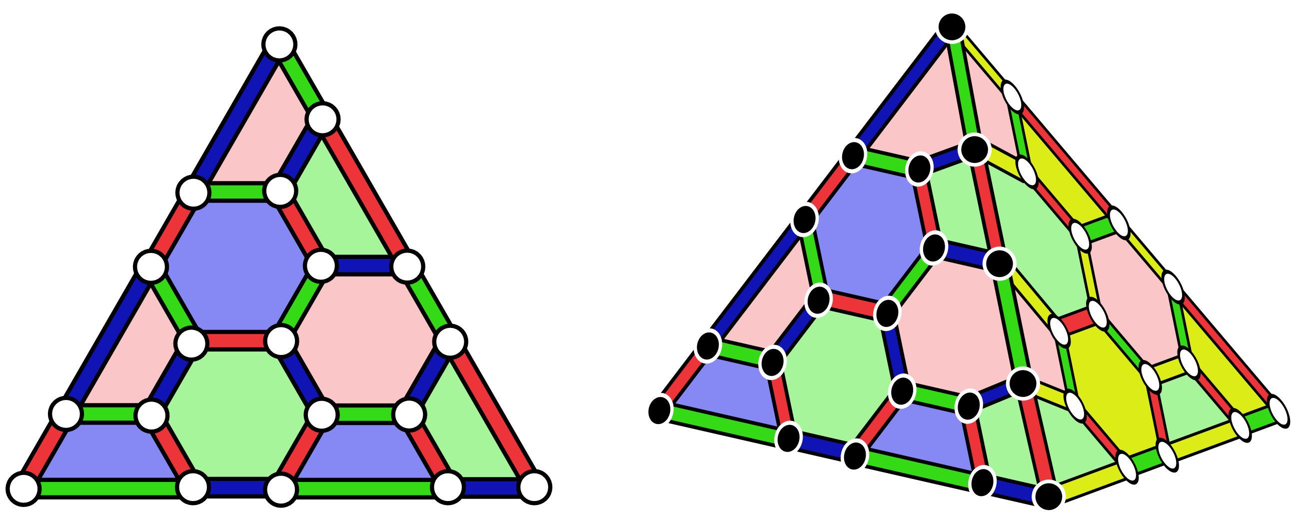
- There is an X and a Z gauge syndrome
- Any of them can be fixed to become part of the stabilizer, but not both!
- Each option corresponds to a *conventional* 3D color code



dimensional jumps

geometry

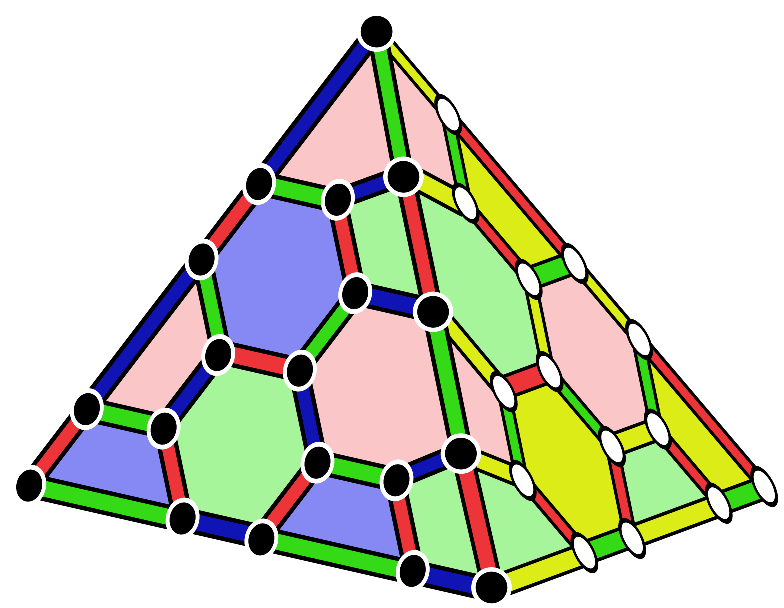
- A facet of a tetrahedral 3D color code lattice is a triangular 2D color code lattice (the *outer* lattice)

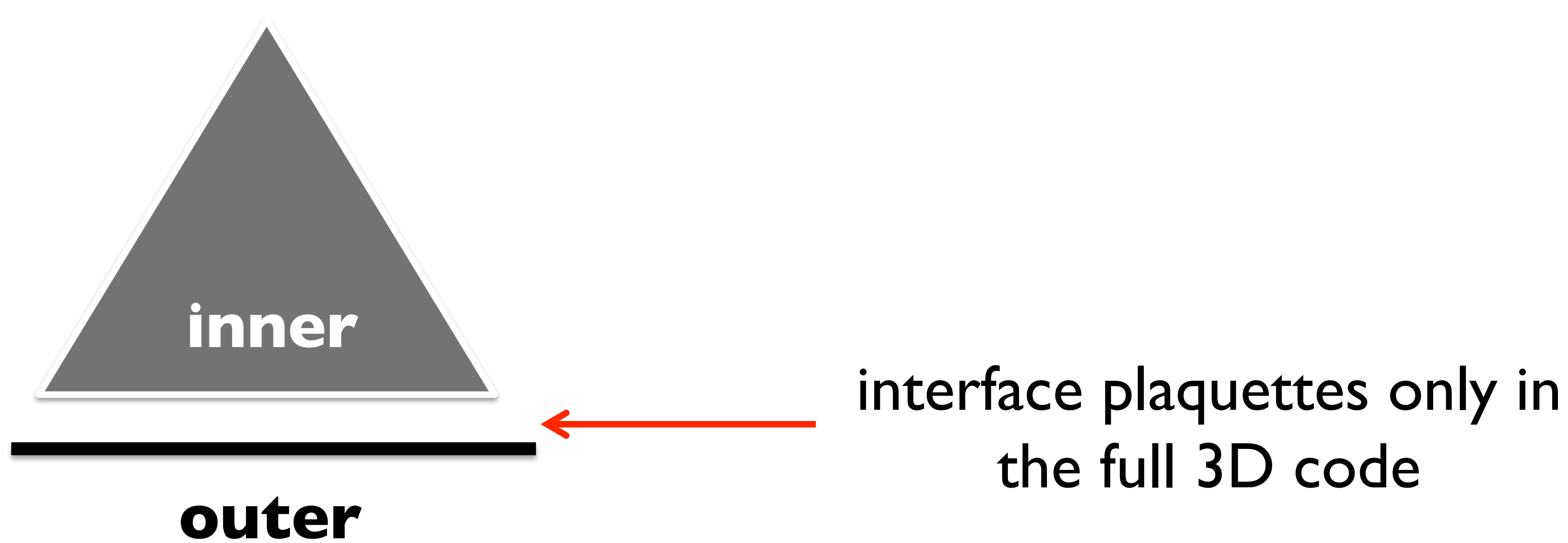


- Gauge generators are plaquette operators in both cases
- X and Z products with support on the whole triangle are logical operators for both

splitting

- *Inner* code:
    - qubits in the inner lattice
    - generators also plaquettes
- Split 3D code in 2D + inner!



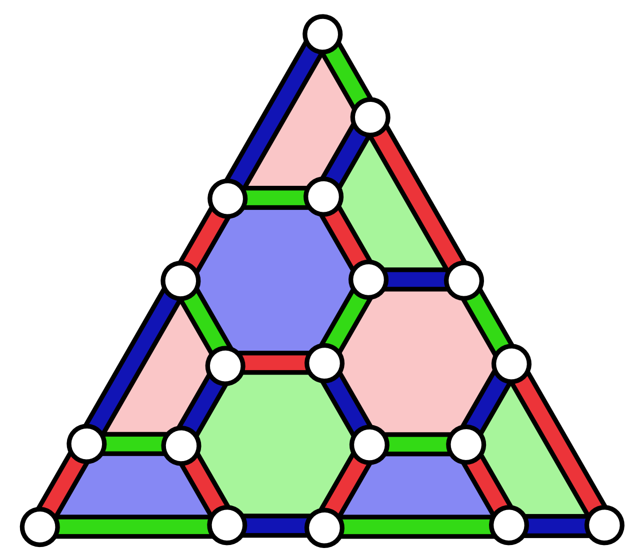


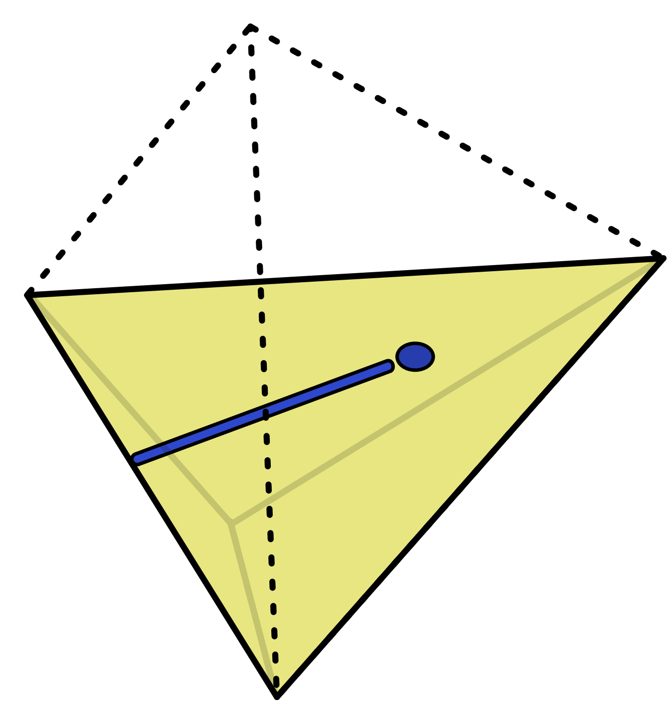
ideal dimensional collapse

① Discard inner qubits

② Measure syndrome of 2D code

③ Apply 'gauge' correction





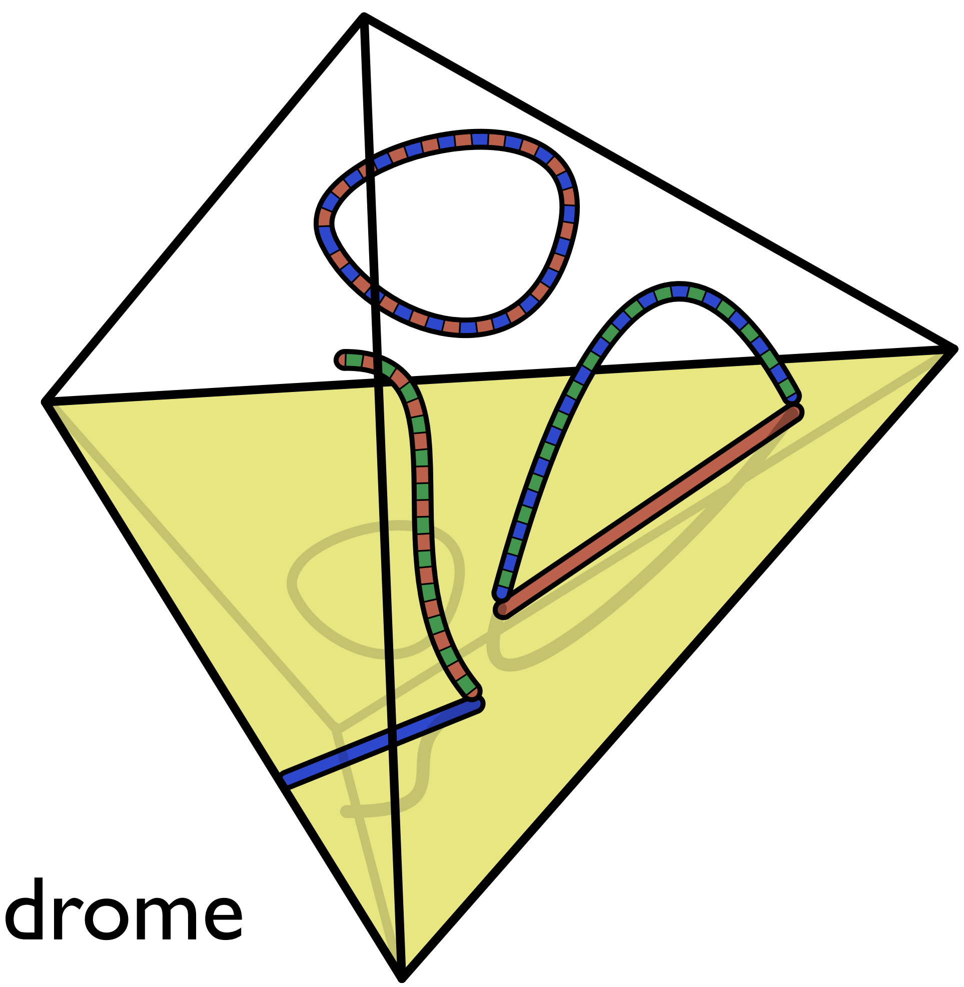
- The correction operator is a product of edge operators (restriction of interface plaquettes)
- This process is **not** fault-tolerant

fault-tolerant dimensional collapse

① Measure inner gauge syndrome

② Discard inner qubits

③ Apply 'gauge' correction



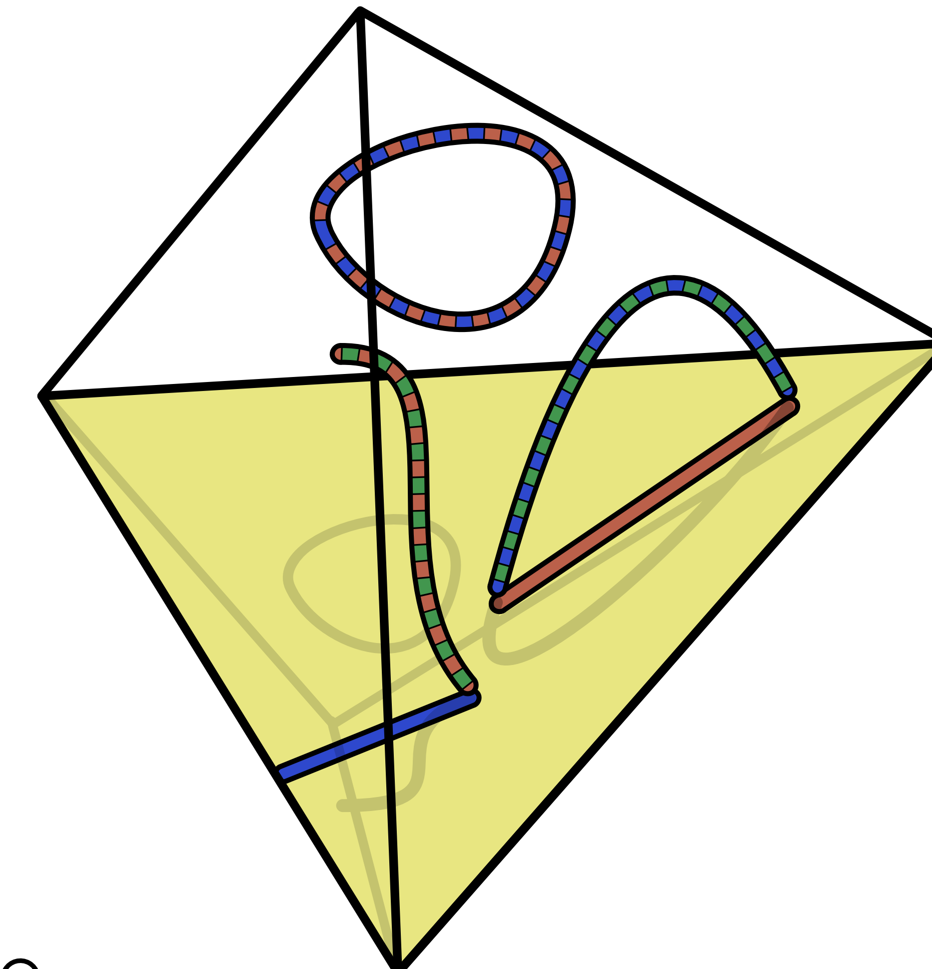
- syndrome = endpoints of gauge syndrome
- This process tolerates measurement errors:
  - string shorter than flux-line
  - string and flux connected

fault-tolerant dimensional collapse

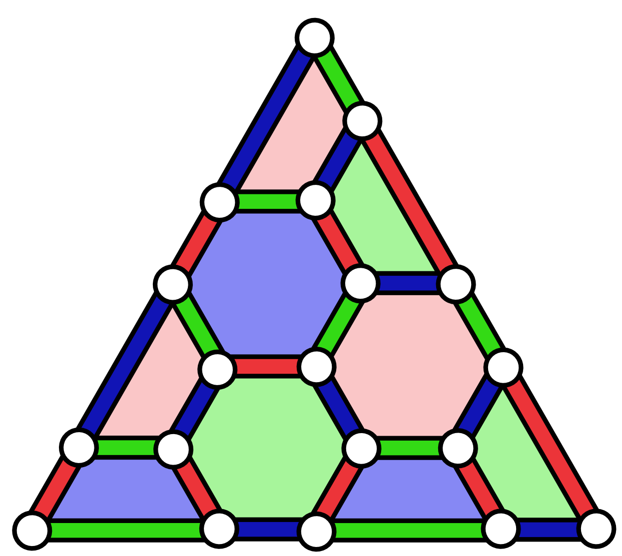
① Destructive measurement of

rg, gb and rb plaquette ops

② Apply 'gauge' correction



- The rest of flux types cannot have endpoints at a rgb-facet

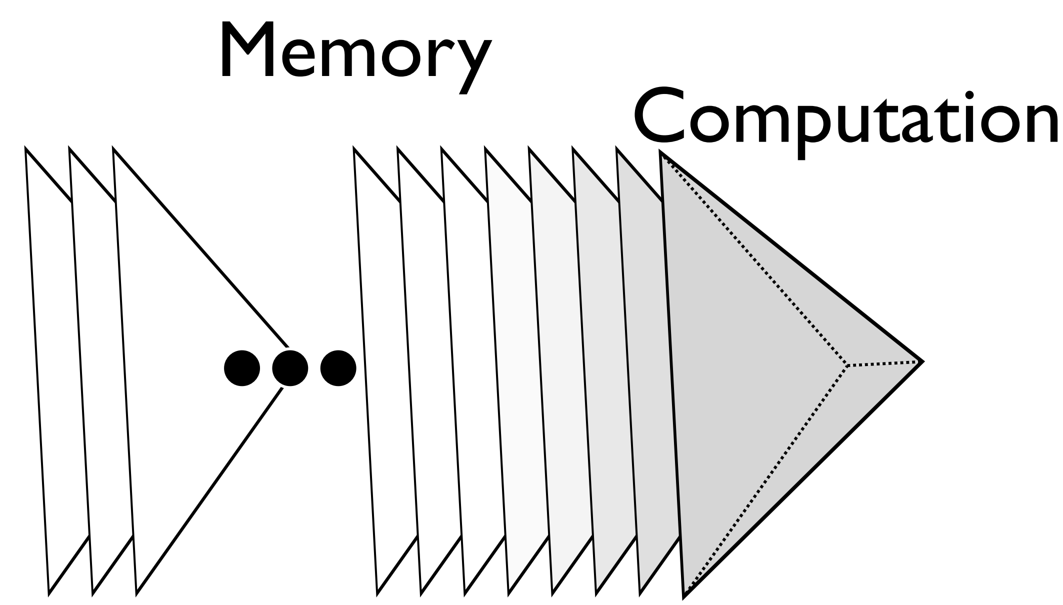


fault-tolerant dimensional jumps

- Dimensional collapse is fault-tolerant:
  - Noise in outer qubits, if local, remains local
  - Local noise in inner qubits prior to measurement can be absorbed as local measurement noise.
- The inverse dimensional jump is fault-tolerant:
  - initialize the inner color code with single-shot error correction.

3D-local layout

- Memory:
  - stack of 2D color codes
  - shuffling via transversal swap
  - error tracking
- Computation:
  - Pair of 2D color codes + 3D structure of inner code
  - Fault-tolerant quantum-local universal operations



Higher dimensions

discussion

- Error thresholds?
- What are the limitations in 2D?
- What about non-geometrical locality?
- Could there be related *3D* self-correcting systems?

$$H = -\sum_{g \in \mathcal{G}_0} J_g \, g$$

the end?