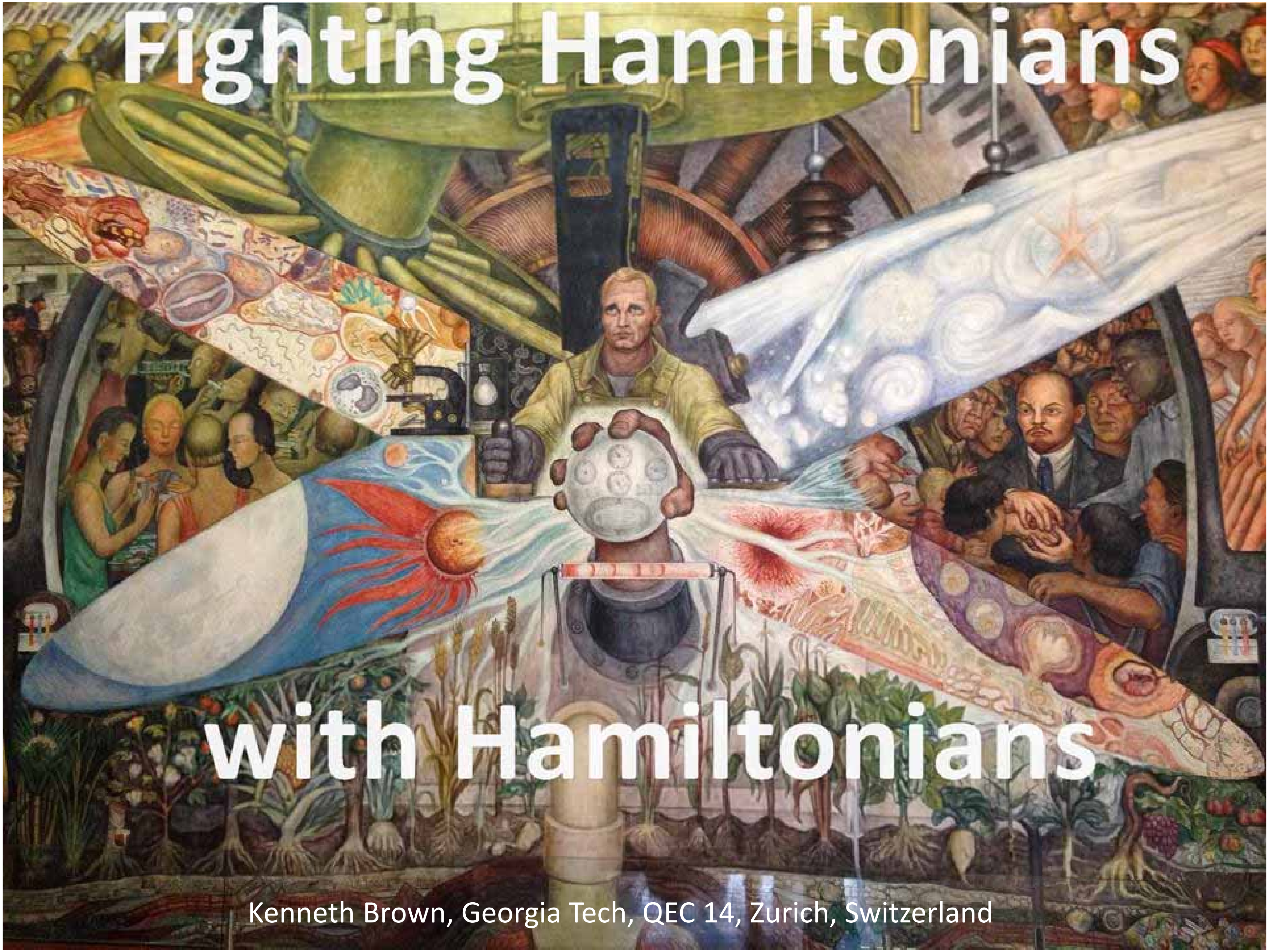
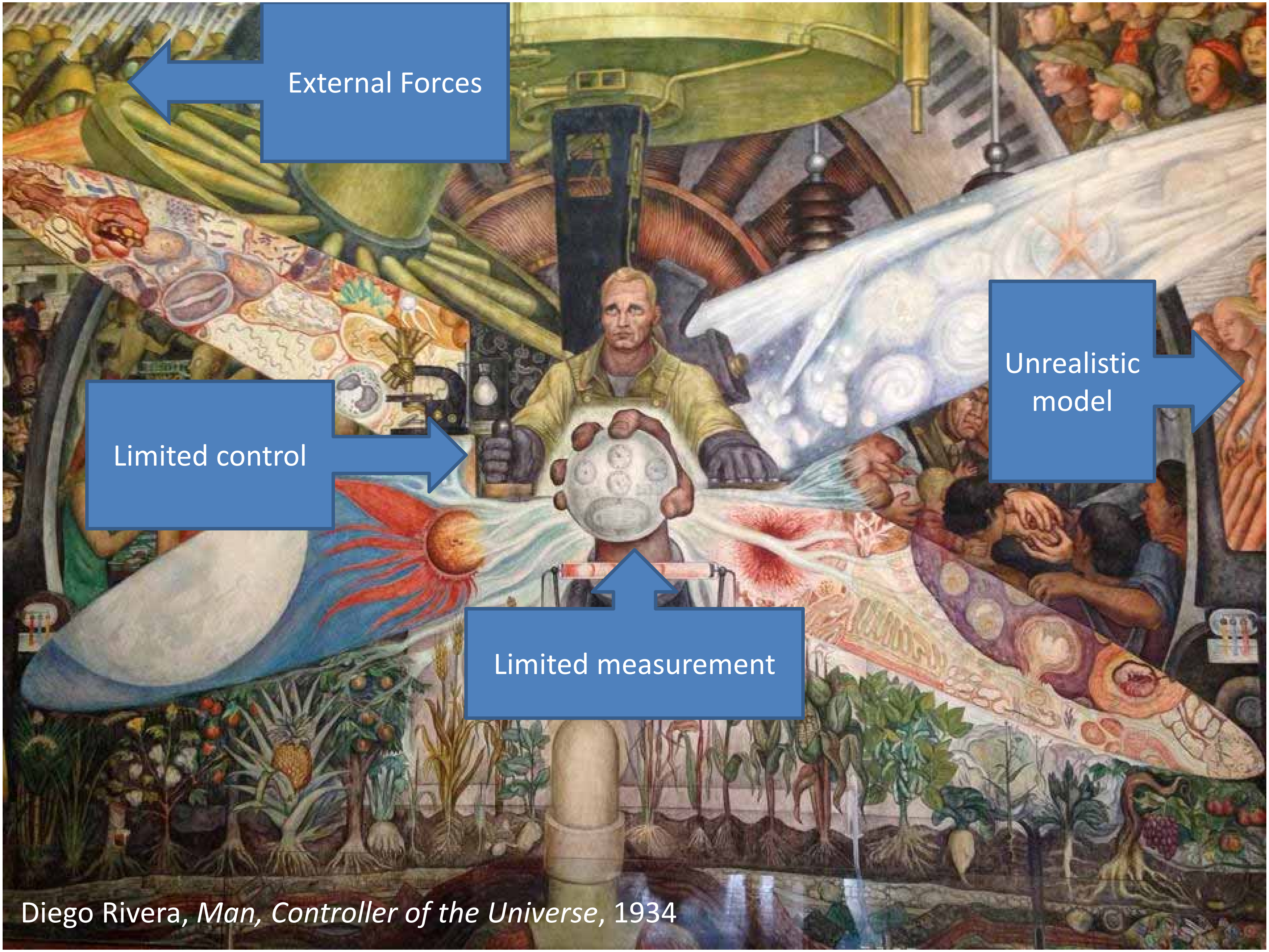


Fighting Hamiltonians with Hamiltonians

Kenneth Brown, Georgia Tech, QEC 14, Zurich, Switzerland

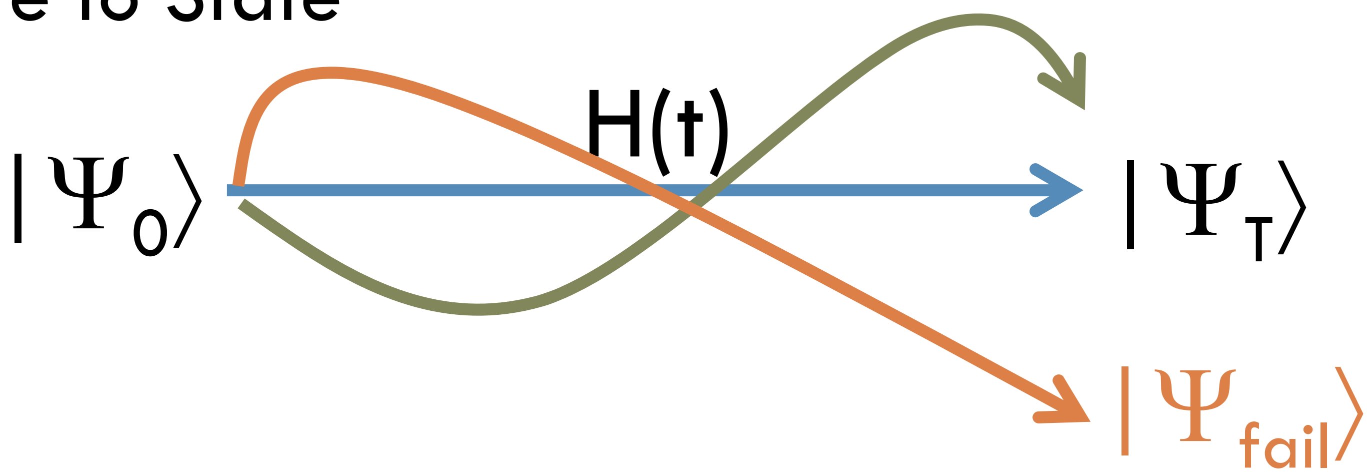


Diego Rivera, *Man, Controller of the Universe*, 1934

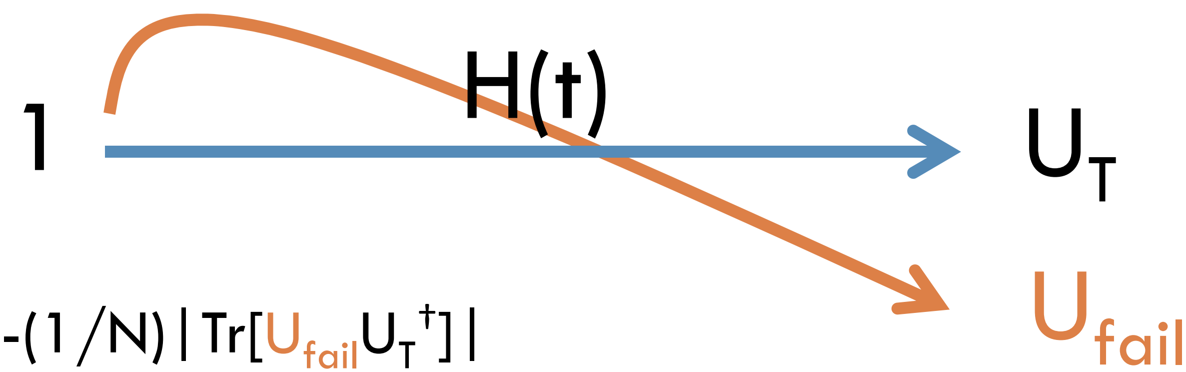
What does it mean to control a quantum system?

State to State or Unitary Evolution

State to State



Unitary Evolution



Error $= 1-(1/N)|\mathrm{Tr}[U_{fail}U_T^{\dagger}]|$

Gates are built from Hamiltonians

Unitary evolution is generated by Hamiltonians

$$
\begin{aligned}
U &= \exp\left[-iHt\right] \\
U &= \mathcal{T}\exp\left[-i\int_0^t H(t')dt'\right]
\end{aligned}
$$

Many different paths in H space lead to the same U

$\exp[-iZt]=\exp[-iZ(t+2\pi)]$

$\exp[-iZt]=\exp[-iX\pi/2]\exp[-iYt]\exp[iX\pi/2]$

The problems

1. You have limited control over your Hamiltonian
    1. Limited calibration
    2. Limits on the on and off values of the field
    3. Limits on the switching speeds
    4. Limited ability to keep track of time
2. The outside world is applying an additional Hamiltonian to your system
    1. Classical environment
    2. Quantum environment

Four types of Hamiltonians

$H_0(t)$: the ideal control

$H_e(t)$: the error from limited control

$H_c(t)$: the error from a classical environment

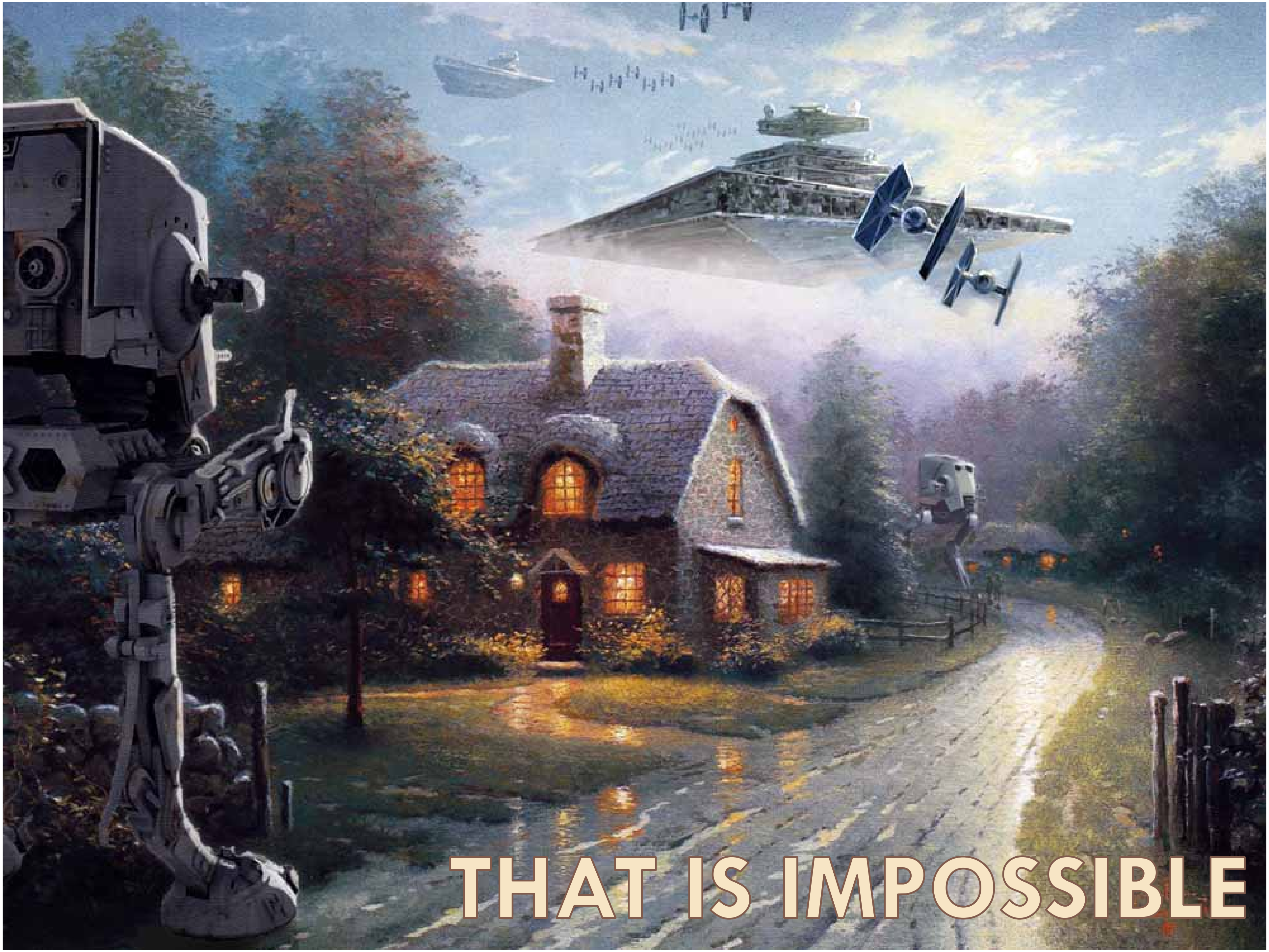
$H_q(t)$: the error from a quantum environment

$H(t)=H_0(t)+H_e(t)+H_c(t)+H_q(t)=H_0(t)+H_b(t)$

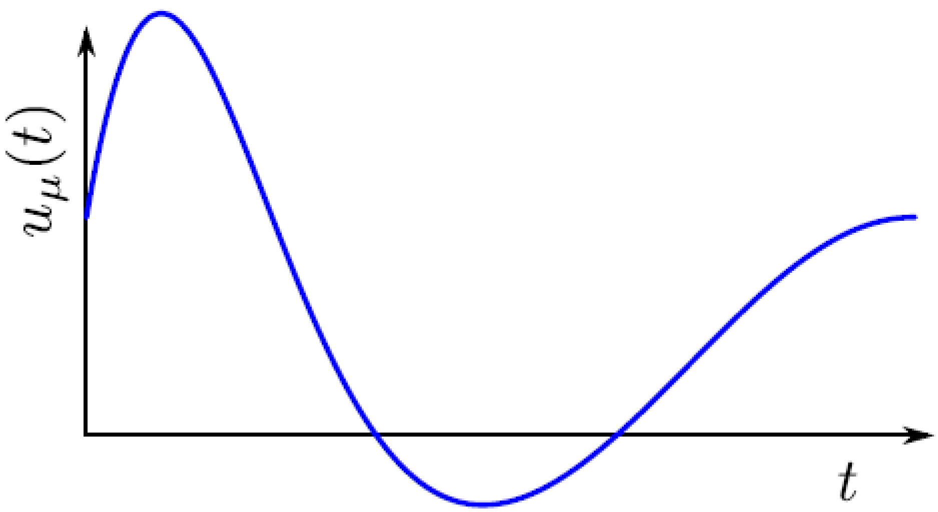
Goal

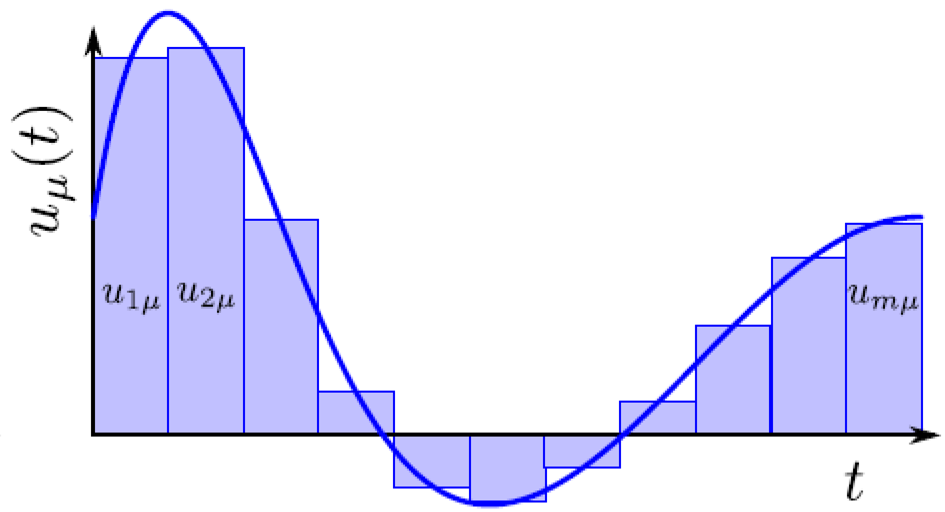
For a given U, find $H_0(t)$ such that

$$U = \mathrm{T}\exp\left(-i\int_{t_0}^{t_1} H_0(t')dt'\right) = \mathrm{T}\exp\left(-i\int_{t_0}^{t_1} H(t')dt'\right)$$


THAT IS IMPOSSIBLE

Limit to pulses



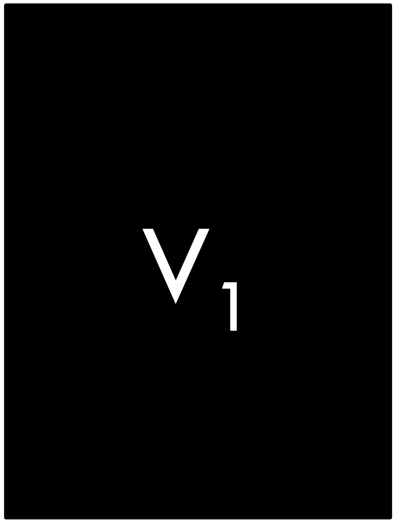


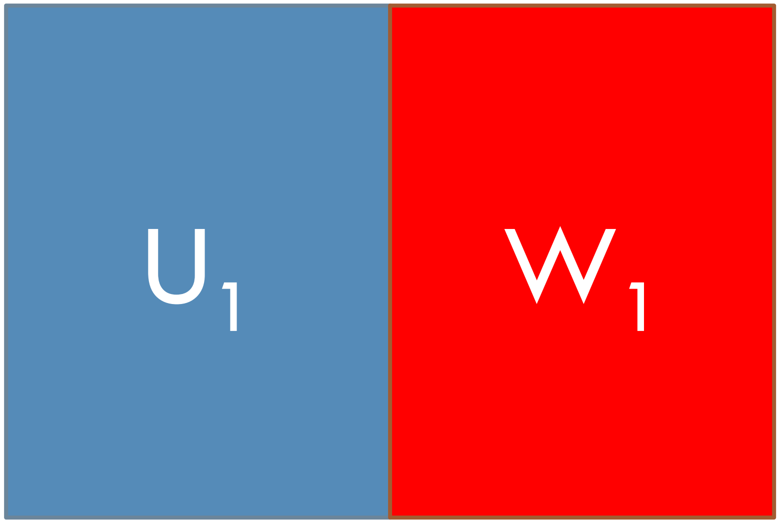
Consider the control Hamiltonian as a sum of time-independent Hamiltonians

$H_0(t)=\Sigma\ u_\mu(t)H_\mu$

Instead of continuously changing the constants we can imagine discrete steps.

Separate the good from the bad



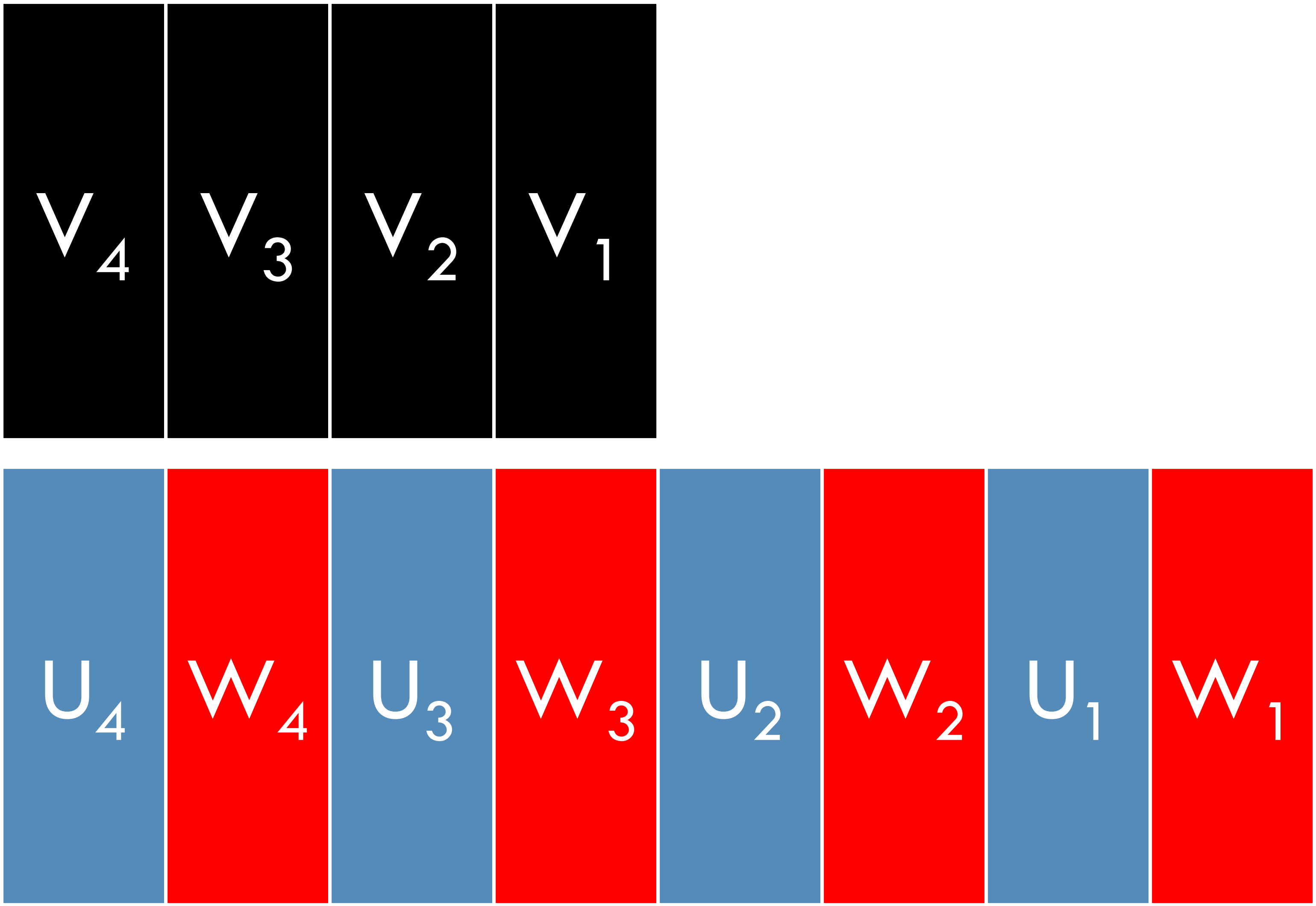


$$
\begin{aligned}
V_1 &= \mathcal{T}\exp\left[-i\int_{t_0}^{t_1} H(t')dt'\right] \\
&= \mathcal{T}\exp\left[-i\int_{t_0}^{t_1} \{H_{0,1} + H_b(t')\}\, dt'\right] \\
&= U_1 W_1
\end{aligned}
$$

$$
\begin{aligned}
U_1(t) &= \exp\left[-iH_{0,1}(t-t_0)\right] \\
U_1 &= U_1(t_1)
\end{aligned}
$$

$$
W_1 = \mathcal{T}\exp\left[-i\int_{t_0}^{t_1} U_1^\dagger(t')H_b(t')U_1(t')dt'\right]
$$

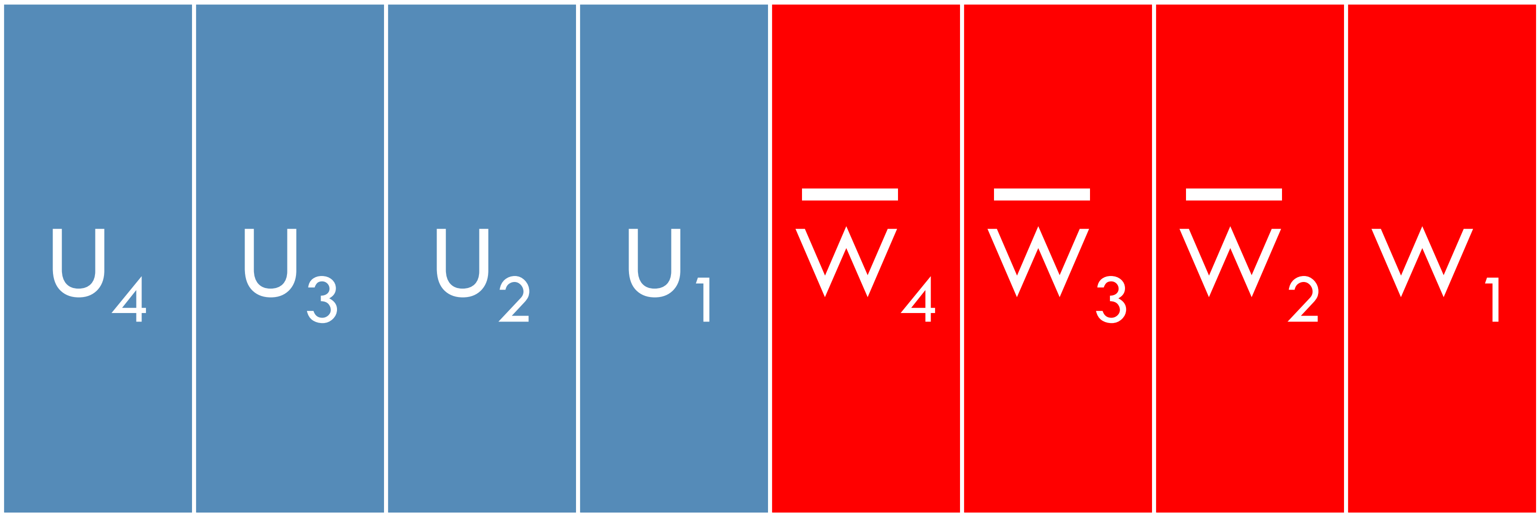
Separate the good from the bad



Separate the good from the bad

U_4	W_4	U_3	W_3	U_2	W_2	U_1	W_1
U_4	W_4	U_3	W_3	U_2	U_1	$\overline{W}_2$	W_1

Separate the good from the bad



$$
\begin{aligned}
W_k &= \mathcal{T} \exp\left[-i \int_{t_0}^{t_1} U_k^\dagger(t') H_b(t') U_k(t') dt'\right] \\
\overline{W}_k &= U_1^\dagger ... U_{k-2}^\dagger U_{k-1}^\dagger W_k U_{k-1} U_{k-2} ... U_1
\end{aligned}
$$

Good news:
W's are changed by U's

Can we make the product of W's approximate I?


THAT IS IMPOSSIBLE


Bounded Strength
Slowly Varying

Doing nothing as best one can

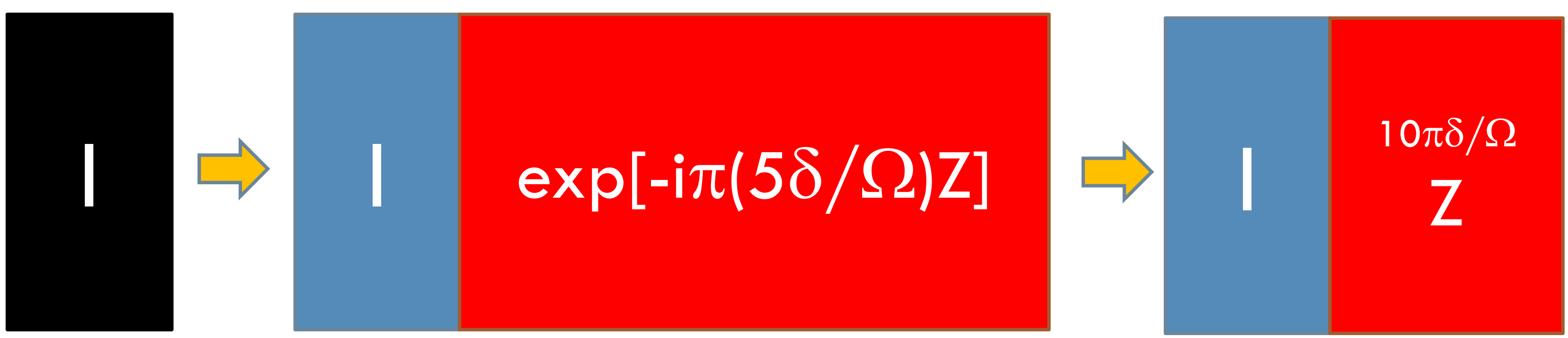
- Control Hamiltonian

$$H_0 = \frac{1}{2}(\Omega_x X + \Omega_y Y)$$

- Error Hamiltonian

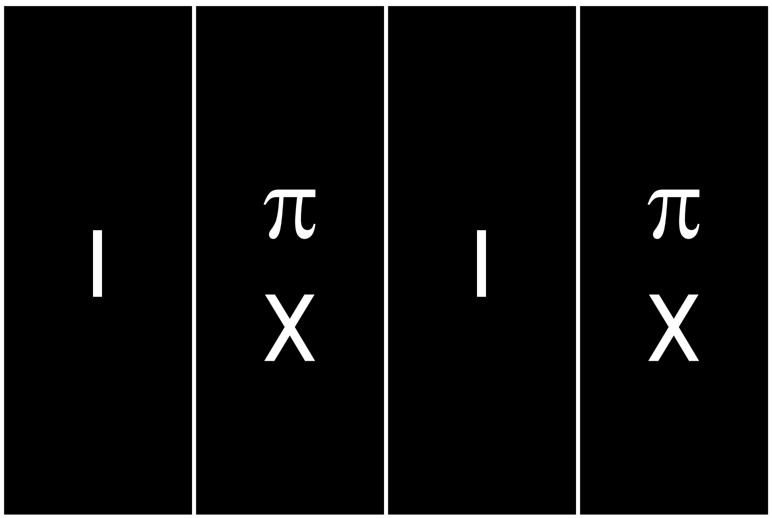
$$H_q = \frac{\delta}{2} Z$$

- Goal: Perform the Identity gate in a time $10\pi/\Omega$



Spin Echo

- We can change the sign of the error Hamiltonian by applying π rotations about X. XZX=-Z



Hahn, Phys. Rev. (1950)

Spin Echo

- We can change the sign of the error Hamiltonian by applying π rotations about X. $XZX=-Z$

I	α Z	π X	β $\eta_{x,y}$	I	α Z	π X	β $\eta_{x,y}$

$\alpha=4\pi\delta/\Omega$ $\quad$ $\beta\approx\delta/\Omega$

$\eta_{x,y}=\eta_0 X+\eta_1 Y$

$\eta_{z,x}=\eta_0 Z+\eta_1 X$

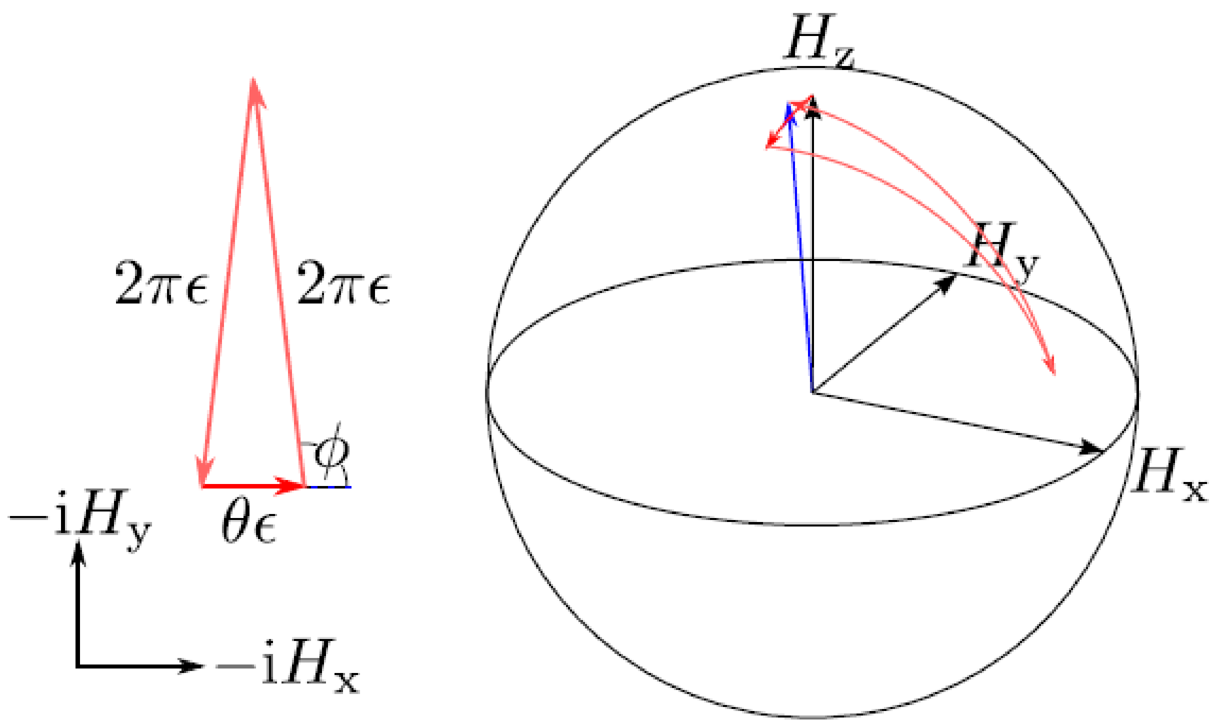
Spin Echo

- Push the errors to the end

I	π X	I	π X	α Z	β $\eta_{x,-y}$	$-\alpha$ Z	β $\eta_{x,y}$

Operators do not commute

Small Hamiltonians add

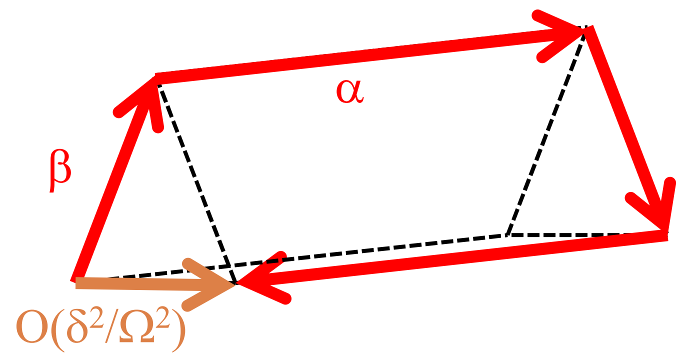


- Lie group U(N) generated by the Lie algebra u(N)
- Some elements in u(N) do not commute.
- The space has curvature but is locally flat.

Spin Echo

I	π X	I	π X	α Z	β $\eta_{x,-y}$	$-\alpha$ Z	β $\eta_{x,y}$

Spin echo reduces the residual error Hamiltonian quadratically in δ/Ω.



Quantum Bath

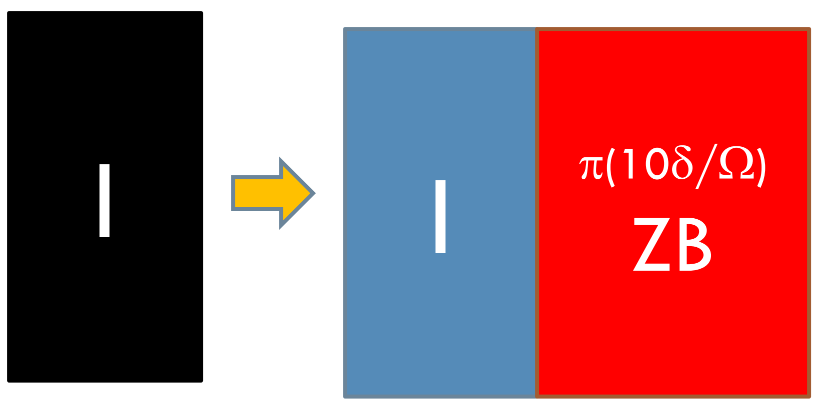
- Control Hamiltonian

$$H_0 = \frac{1}{2}(\Omega_x X + \Omega_y Y)$$

- Error Hamiltonian

$$H_q = \frac{\delta}{2} Z \otimes B$$

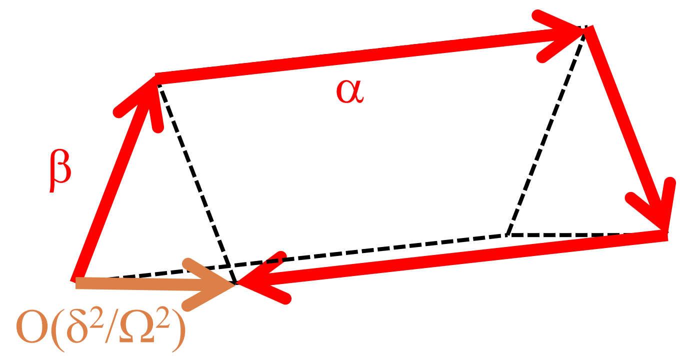
- Goal: Perform the Identity gate in a time $10\pi/\Omega$



Dynamic Decoupling

I	π X	I	π X	α ZB	β $\eta_{x,-yB}$	$-\alpha$ ZB	β $\eta_{x,yB}$

Geometry is the same.
Only difference is the axes labels.



Viola and Lloyd, Phys. Rev. A (1998)
Review: Yang, Wang, Liu arXiv:1007.0623

Environment

- Errors in all directions

$$H_q = \frac{\delta}{2}\sum_i \sigma_i \otimes B_i$$

$$H_q = \frac{\delta}{2}\sum_k S_k \otimes B_k$$

Zanardi, Phys. Rev. Lett. (1999)
Viola, Knill, and Lloyd, Phys. Rev. Lett. (1999)
Khodjasteh and Lidar, Phys. Rev. Lett. (2005)

Can cancel by an appropriate choice of pulses

$$\sum_i U_i^\dagger H_q U_i = 0$$

I	π X	I	π Y	I	π X	I	π Y

Environments

- Errors changing in time

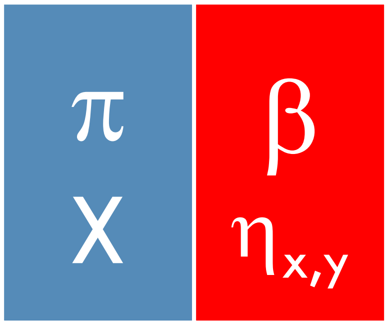
$$H_q(t) = \frac{\delta}{2}\sum_i \sigma_i \otimes B_i(t)$$

$$H_q(t) = \frac{\delta}{2}\sum_k S_k \otimes B_k(t)$$

- Periodic DD amplifies any noise that switches at the pulse period
- Many choices: CDD, UDD, WDD, etc.
- These are all slow noise filters with different properties (next talk: Lorenza Viola)

Khodjasteh and Lidar, Phys. Rev. Lett. (2005); Uhrig, Phys. Rev. Lett. (2007); .Hayes, Khodjasteh, Viola, Biercuk Phys. Rev. A (2011)

Environments and Gates



- Construct sequence that cancels the gate noise
- Dynamically Corrected Gates
- Black-box noise models do not work

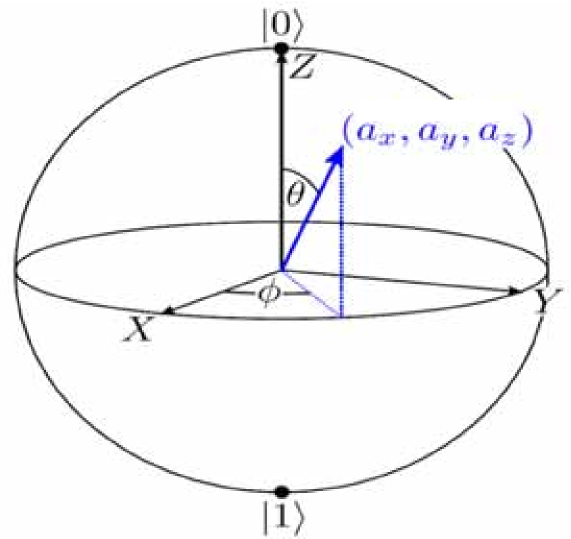
Khodjasteh and Viola, Phys. Rev. Lett.(2009); Phys. Rev. A (2009)

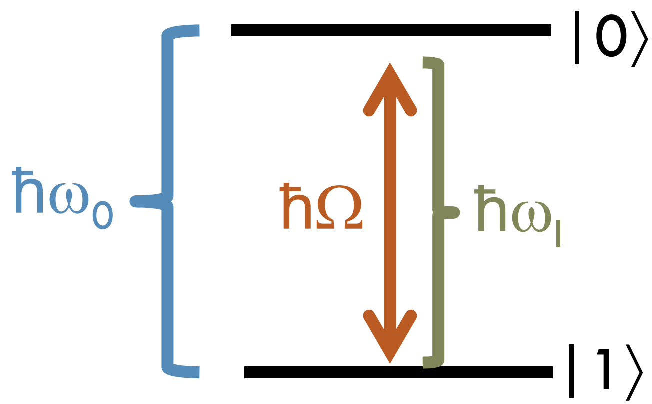
De and Pryadko, Phys. Rev. Lett. (2013); Phys. Rev. A (2014)

Many others


Problems with Control

Control by resonant excitation





Two-level system interacting with an oscillating field

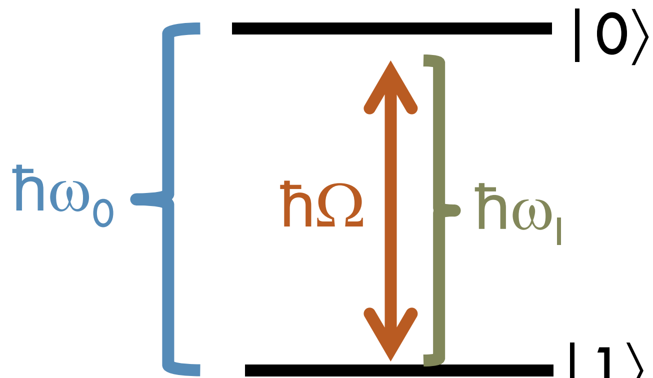
$H=1/2\ [\ \omega_0 Z + \Omega(|0\rangle\langle 1|\exp[-i(\omega_l t+\phi)] + H.c.\ )]$

Switch to the interaction picture $\Delta=\omega_l - \omega_0$

$H_I=1/2\ [\ -\Delta\ Z + \Omega(\cos(\phi)X+\sin(\phi)Y)]$

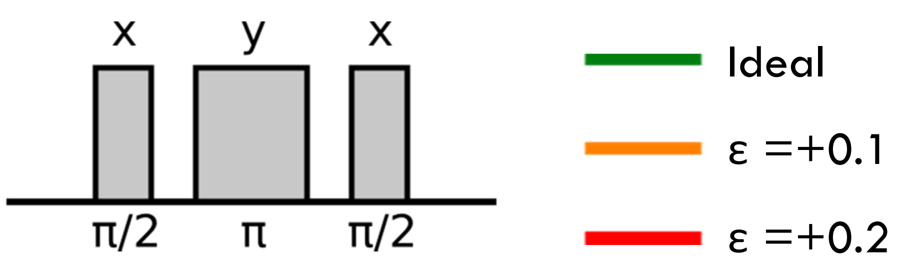
Control errors

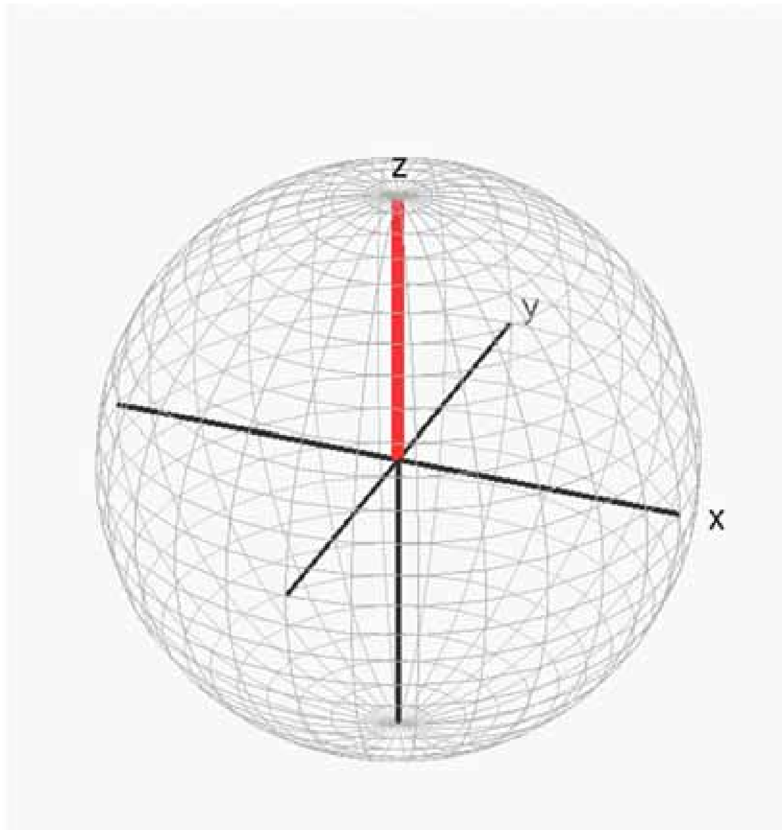
- Errors in Ω
  - Power fluctuations
  - Pointing instability
  - Polarization oscillations
- Errors in $\Delta = \omega_l - \omega_0$
  - Frequency instability of laser
  - Fluctuating magnetic fields
- Errors in ϕ
  - Experimental time relative to local oscillator



Composite pulses

- Initially developed for NMR
- Technique to compensate **systematic errors** in controlling quantum systems
- Can correct **unknown** error
- $\Omega' = \Omega(1+\varepsilon)$

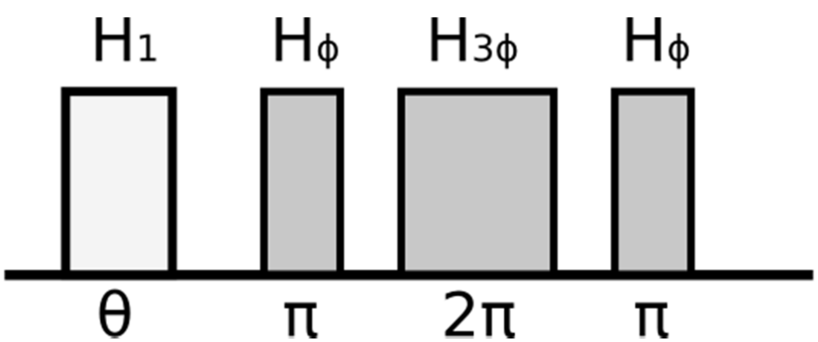




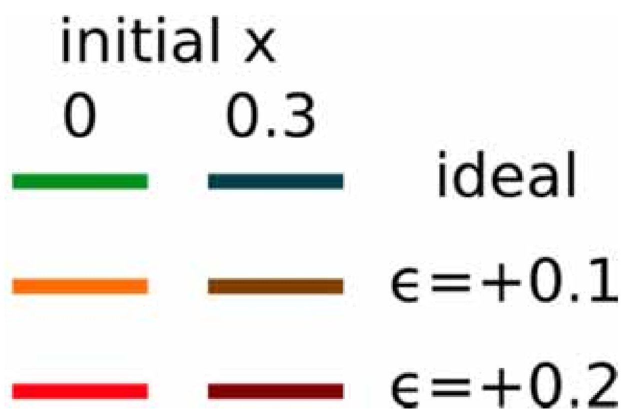
M.H. Levitt and R. Freeman, J. Magn. Reson. **33**, *473* (1979)

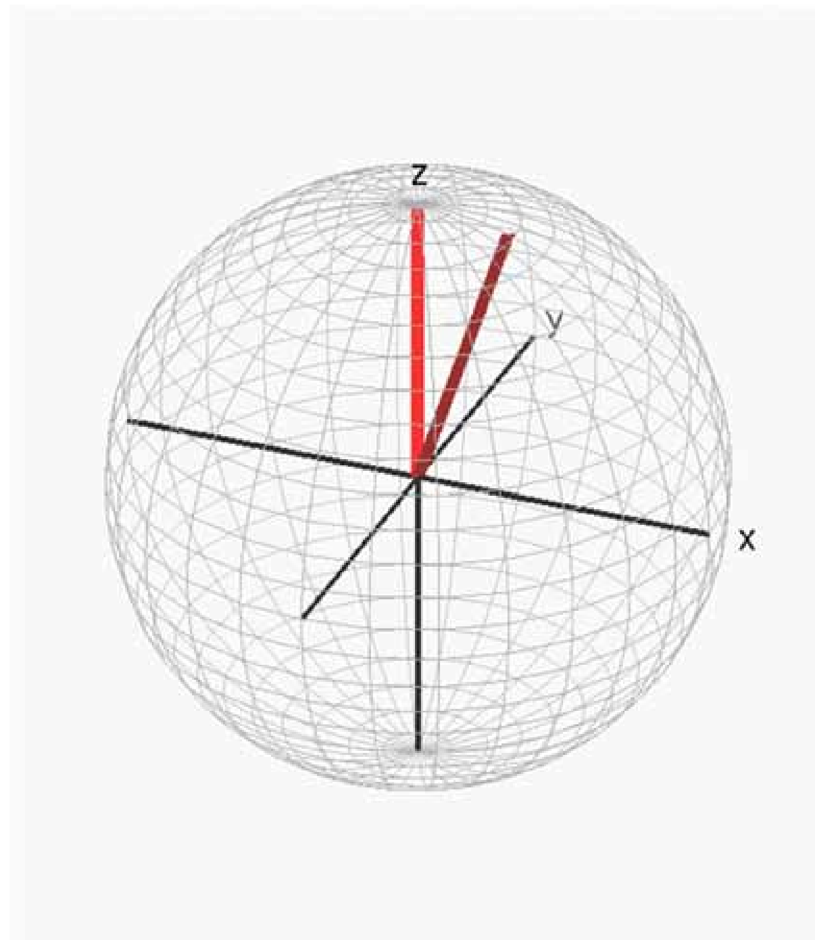
Fully Compensating Pulses

- Example BB1, π/2 rotation about the X axis



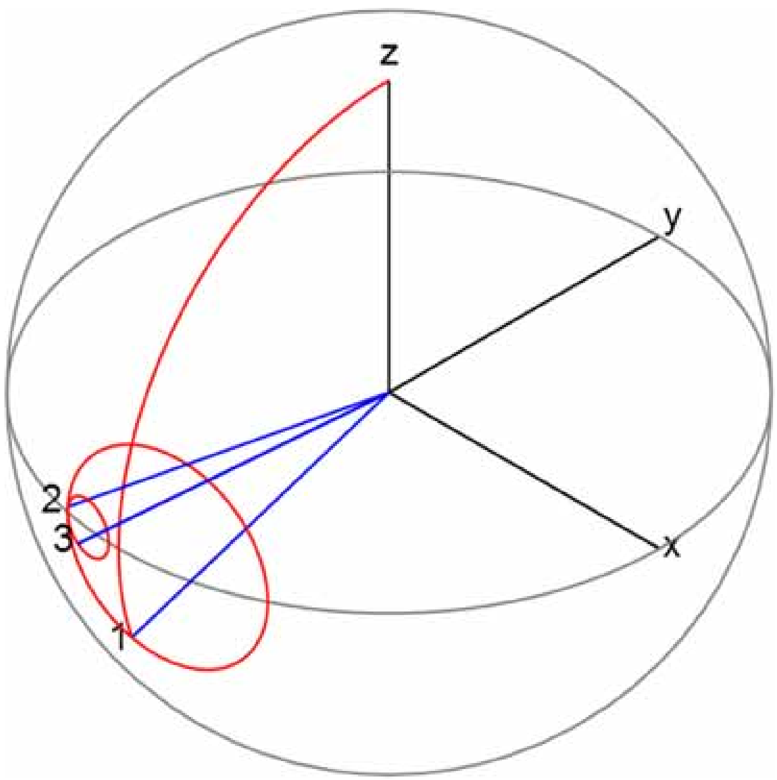
S. Wimperis, J. Magn. Reson. **109**, 221 (1994)



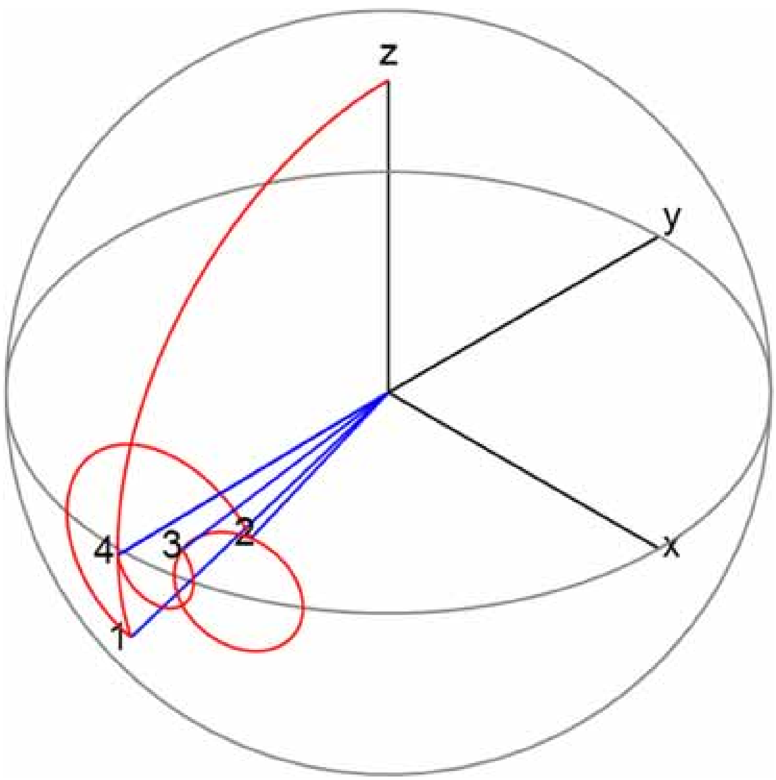


Composite Pulse Sequences

SK1



BB1



Wimperis, J. Magn. Reson. (1994)

KRB, Harrow, and Chuang, Phys. Rev. A **70**,(2004)

Higher order pulses with linear scaling: Low, Yoder, and Chuang (2014)

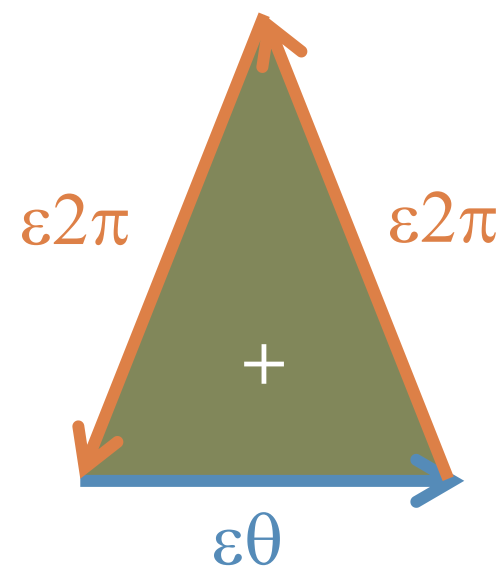
SK1

θ X	2π $\mu_{x,-y}$	2π $\mu_{x,y}$

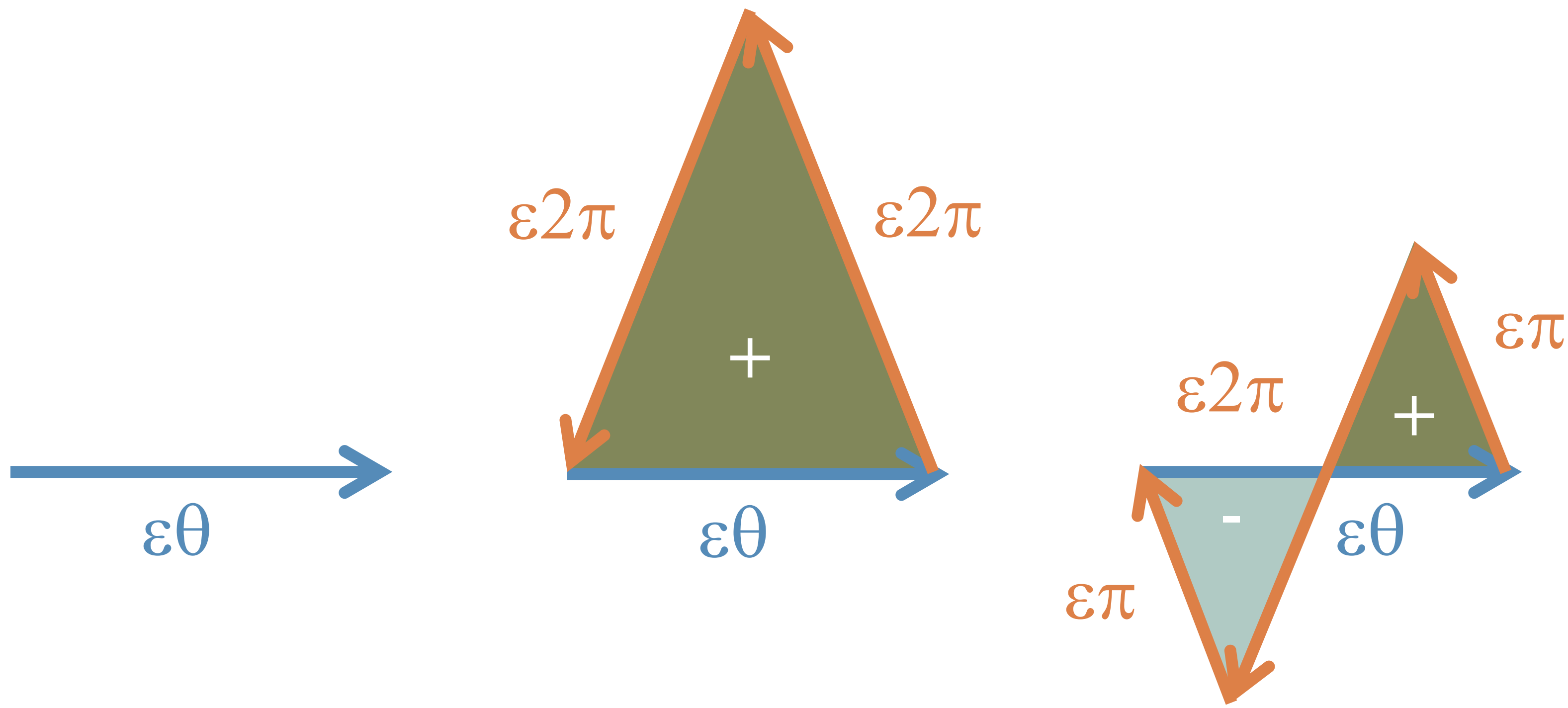
$\mu_{x,y}$=Cos(ϕ)X+Sin(ϕ)Y

Choose ϕ such that Cos(ϕ) =$\theta/(4\pi)$

θ X	2π $\mu_{x,-y}$	2π $\mu_{x,y}$	$\varepsilon\theta$ X	$\varepsilon 2\pi$ $\mu_{x,-y}$	$\varepsilon 2\pi$ $\mu_{x,y}$



Independent of ε

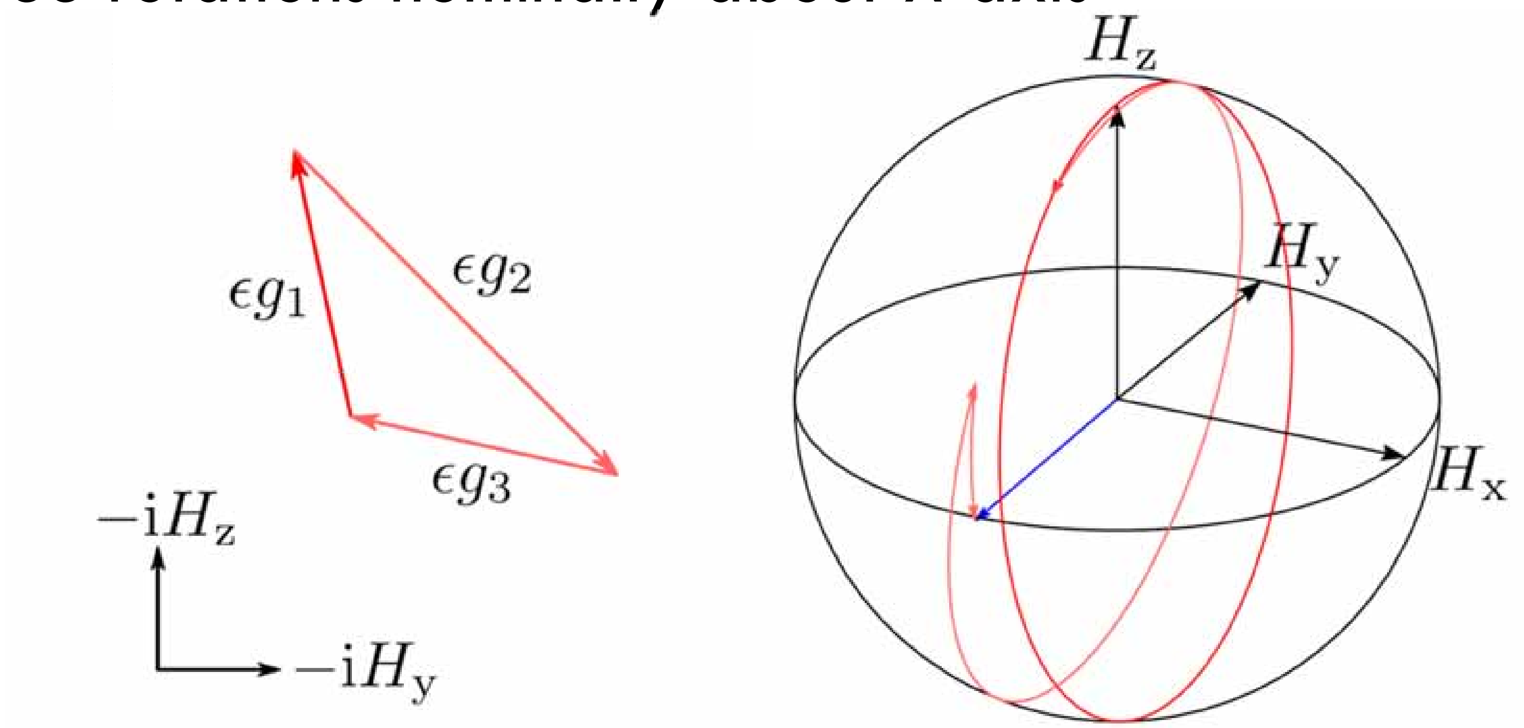


Naïve ~ ε SK1 ~ ε^2 BB1 ~ ε^3

2012 Review: Merrill and KRB, Adv. Chem. Phys. (2014)

CORPSE

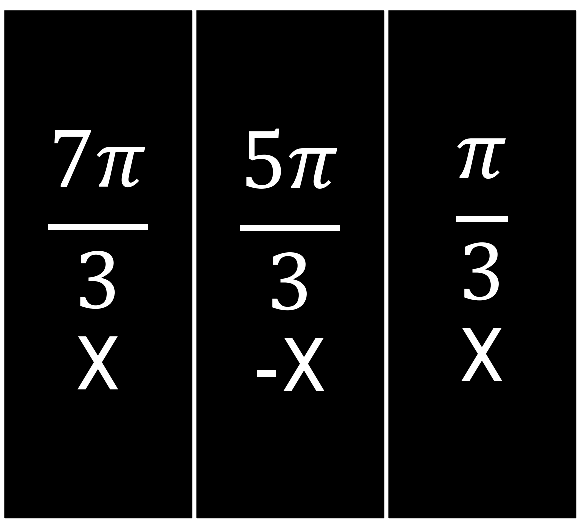
- Fixes detuning errors: $\Delta \rightarrow \Delta(1+\varepsilon)$
- Three rotations nominally about X axis



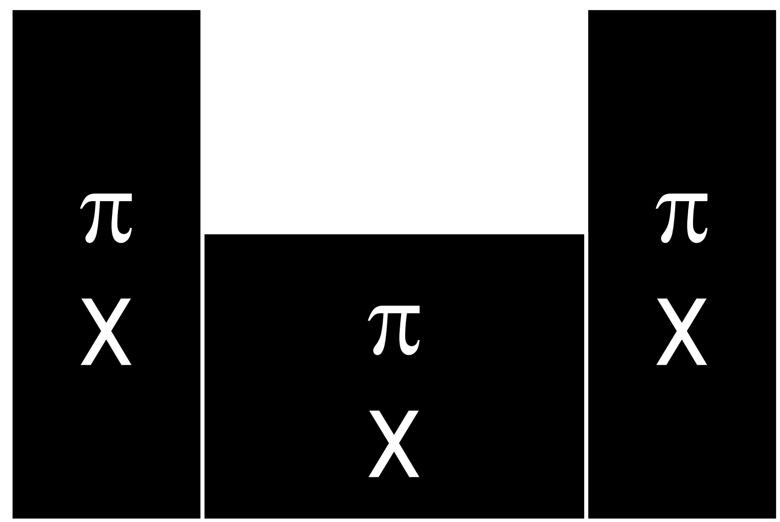
Cummins and Jones, New J. Phys. (2000)
Merrill and KRB, Adv. Chem. Phys. (2014)

Compare to Dynamically Corrected Gates

- Detuning control noise is indistinguishable from an unknown classical field along Z.



Better error suppression at DC
Sensitive to pulse shape
Requires negative control

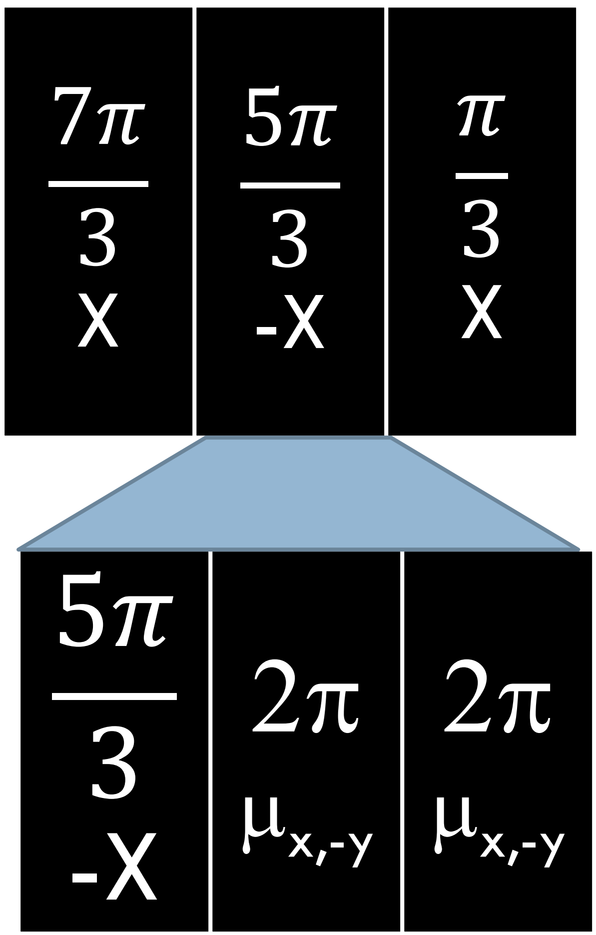


Does not require negative control
Insensitive to pulse shape

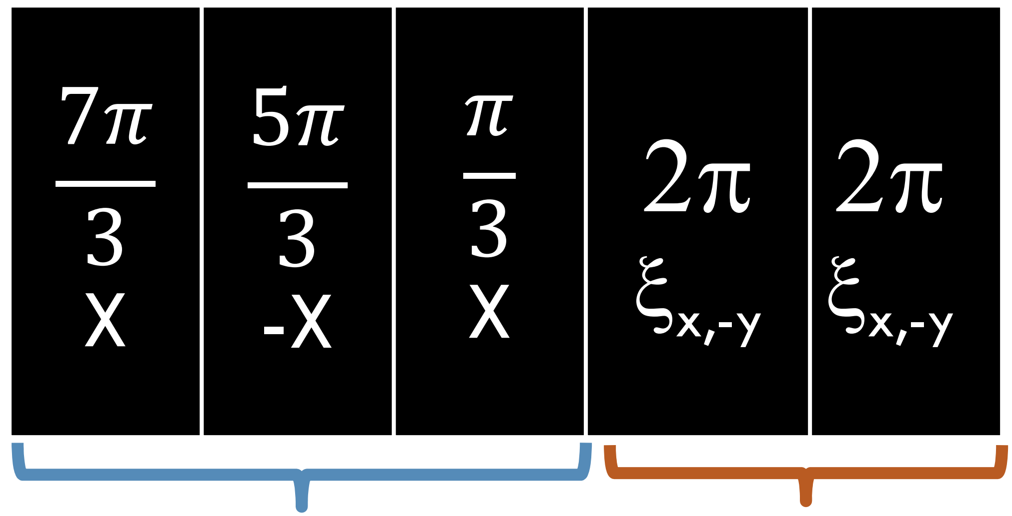
Kabytayev et al. PRA (2014)
Shaped pulses: Pengupta and Pryadko, PRL (2005)

Detuning and Amplitude Errors

Concatenate sequences



Sequences conserve error



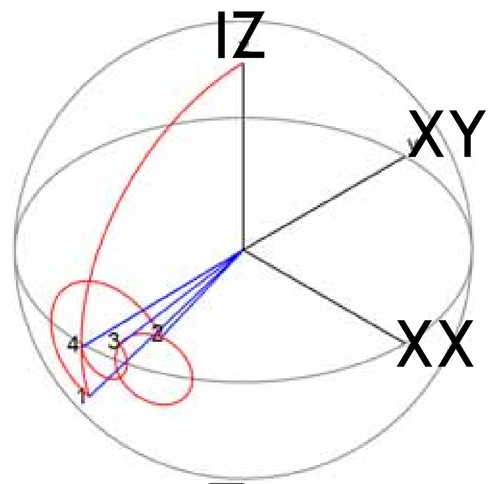
amplitude error same as primitive pulse

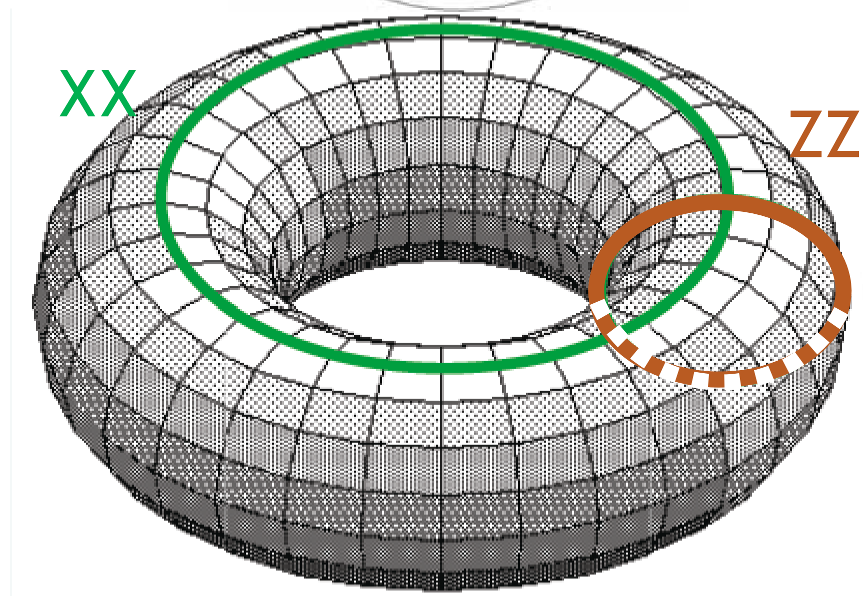
no δ term.

Bando et al., J. Phys. Soc. Jpn. (2013)

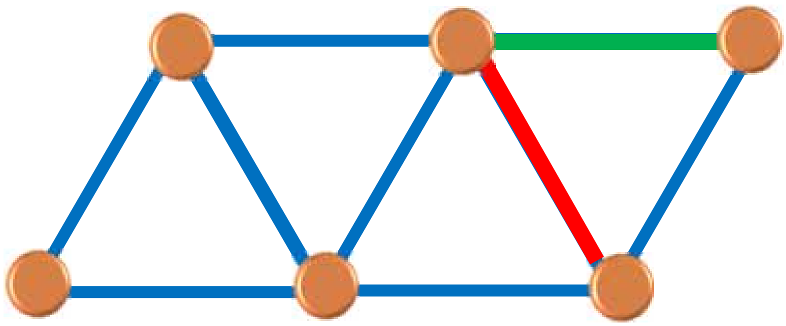
Two Qubits

- Control Algebra (Lie Algebra)
  - $\{I,X,Y,Z\} \otimes \{I,X,Y,Z\}$
- Any two non-commuting operators generate a representation of SU(2)
  - $[XY, IZ]=i2XX$
- No new forms
  - $SU(4)/SU(2) \otimes SU(2)$
  - Algebra: XX, YY, and ZZ





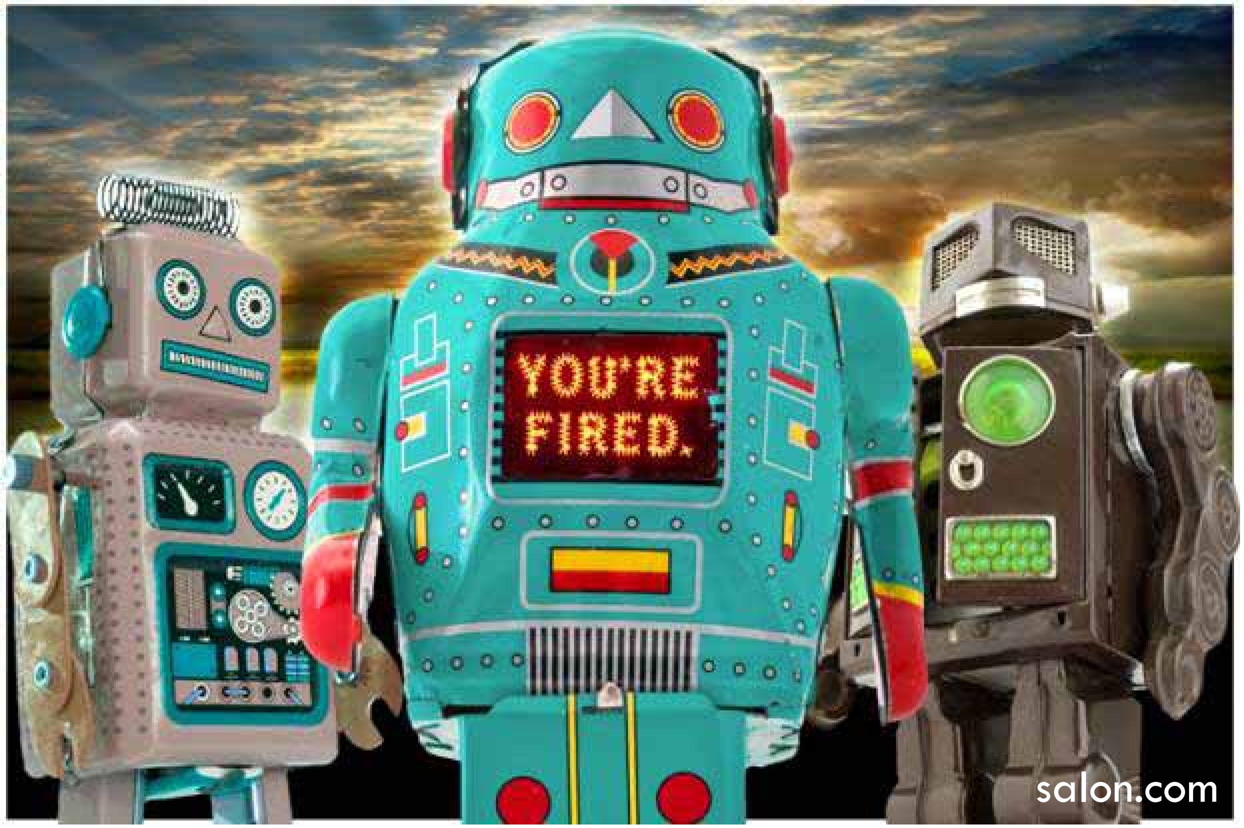
Multi-qubit systems



- Three qubits controlled by XY spin-coupling have compensation sequences equivalent to rotations of a single spin (*XY subalgebra isomorphic to SU(2)*)
- One perfect control can compensate a set of uncorrelated but systematic errors
  - Ising coupled qubits with independent qubit control

Y. Tomita, J.T. Merrill, and KRB
New J. Phys. (2010)

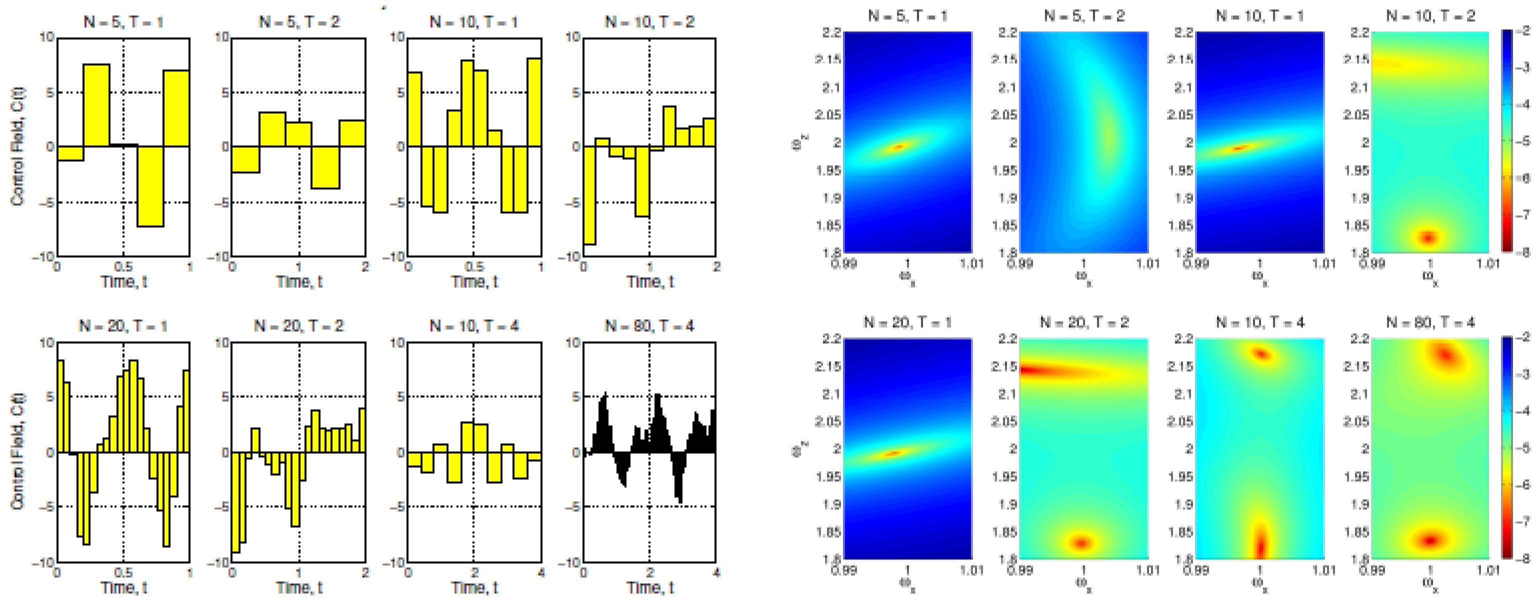
Numerical Quantum Control



Robust and complex

Robust control of quantum gates via sequential convex programming

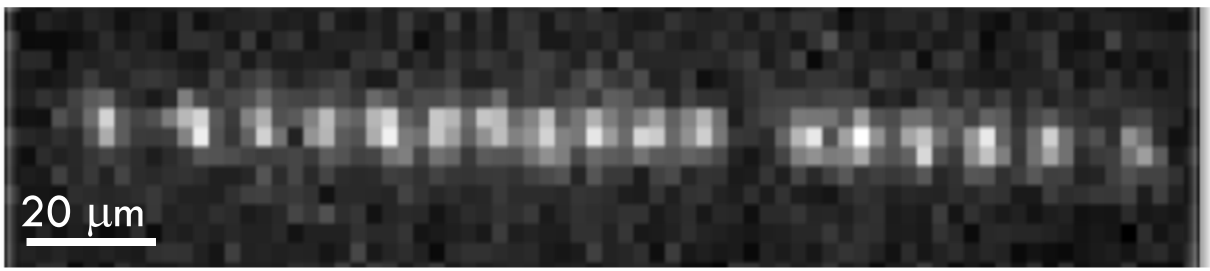
Robert L. Kosut,[1,*] Matthew D. Grace,[2,†] and Constantin Brif[2,‡]

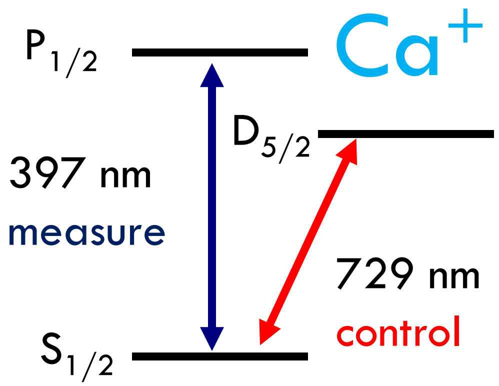


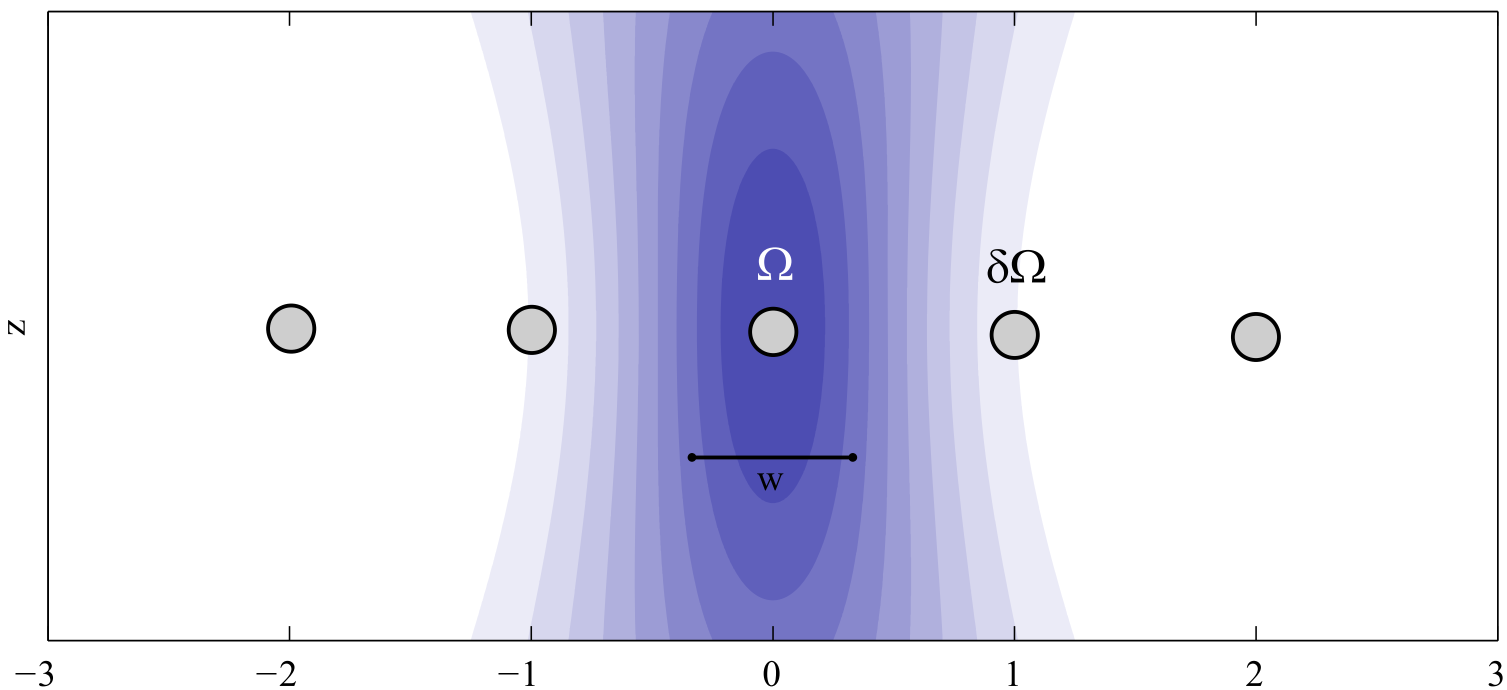
Robust control and robust optimization of uncertain systems are essential in many areas of science and engineering [1–8]. Recently, there has been much interest in achieving robust control of quantum information systems in the presence of uncertainty [9–40]. An important property of quantum information processing that distinguishes it from most other applications is the requirement of an unprecedented degree of precision in controlling the system dynamics. Also, due

Phys. Rev. A **88**, 052326 (2013)

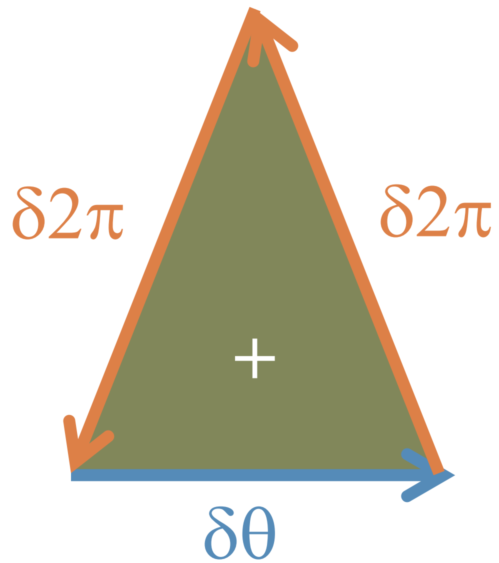
Numerical and Analytical: Addressing Single Ions

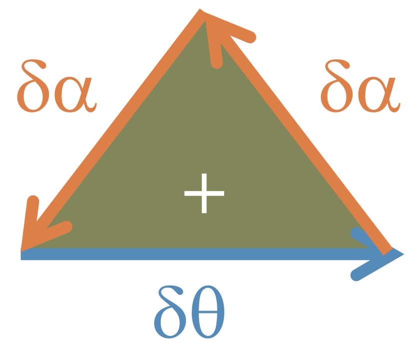






Narrowband sequences





SK1 ~ δ^2

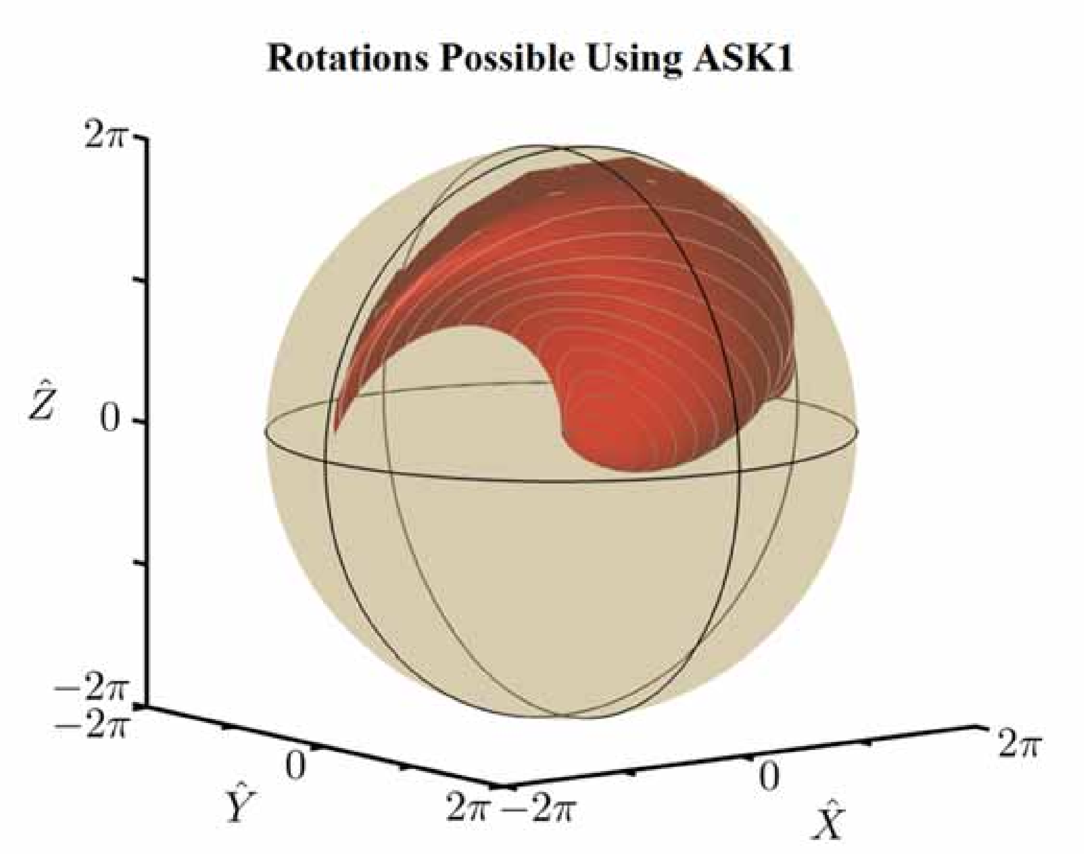
ASK1 ~ δ^2

$R(2\pi,\phi)R(2\pi,-\phi)R(\theta,0)$

$R(\alpha, \xi)R(\alpha,-\xi)R(\theta,0)$

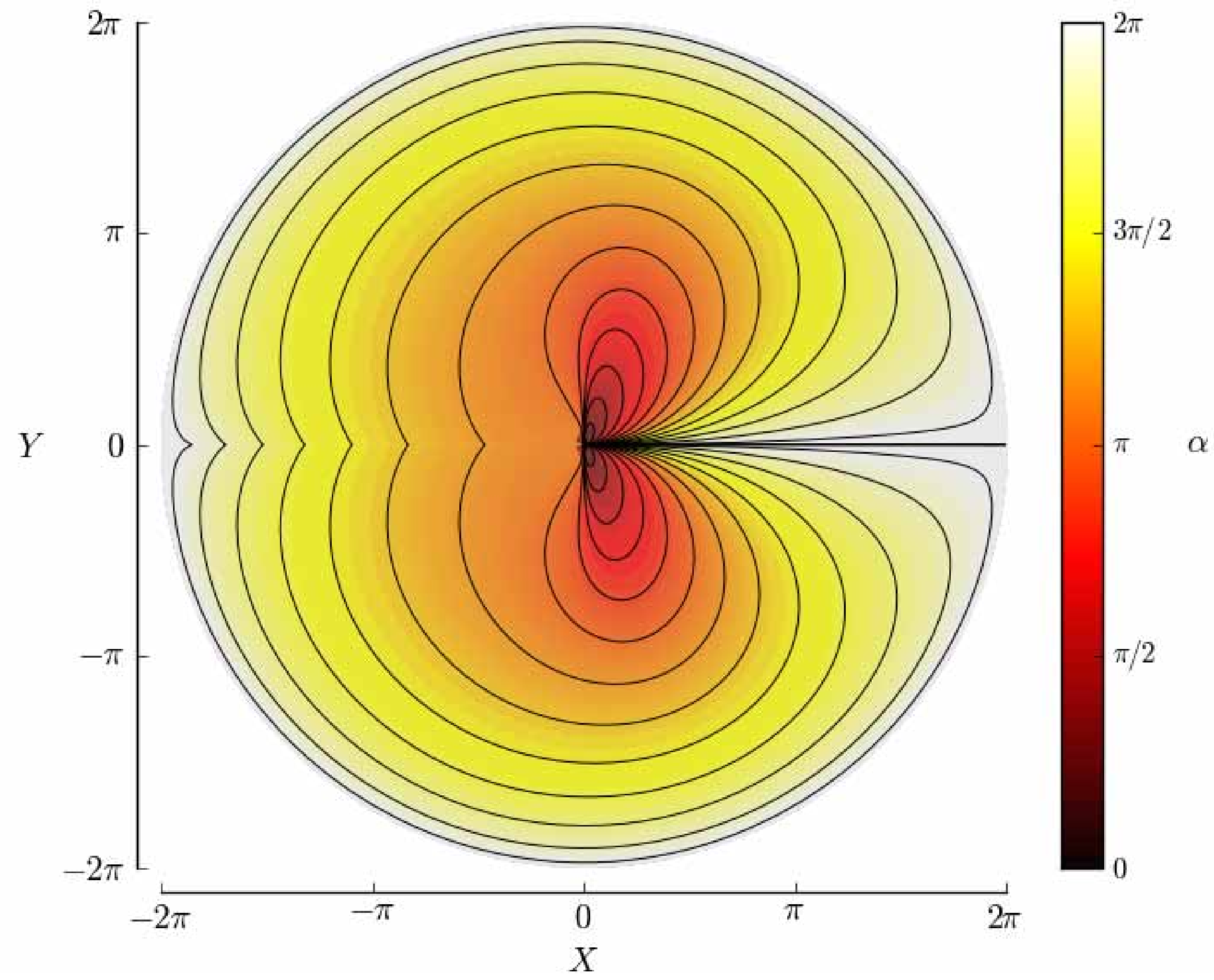
New Composite Pulse Sequence

- ASK1 (3 pulses) reduces crosstalk but generates a different rotation
- TASK1 transforms ASK1 rotations to rotations about axes in x-y plane (5 pulses)

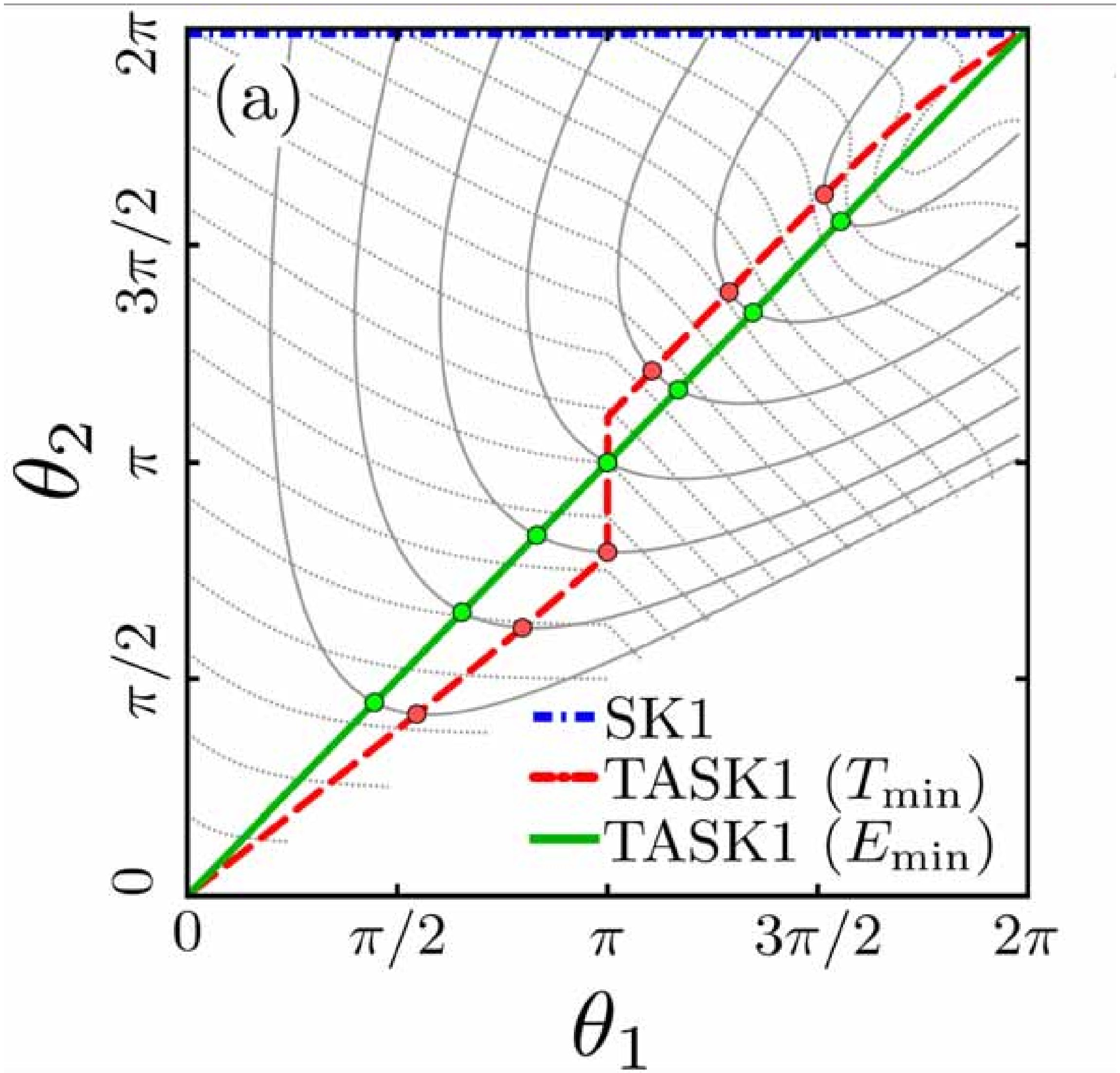


$R=\exp(-i\ \eta\cdot\sigma)$

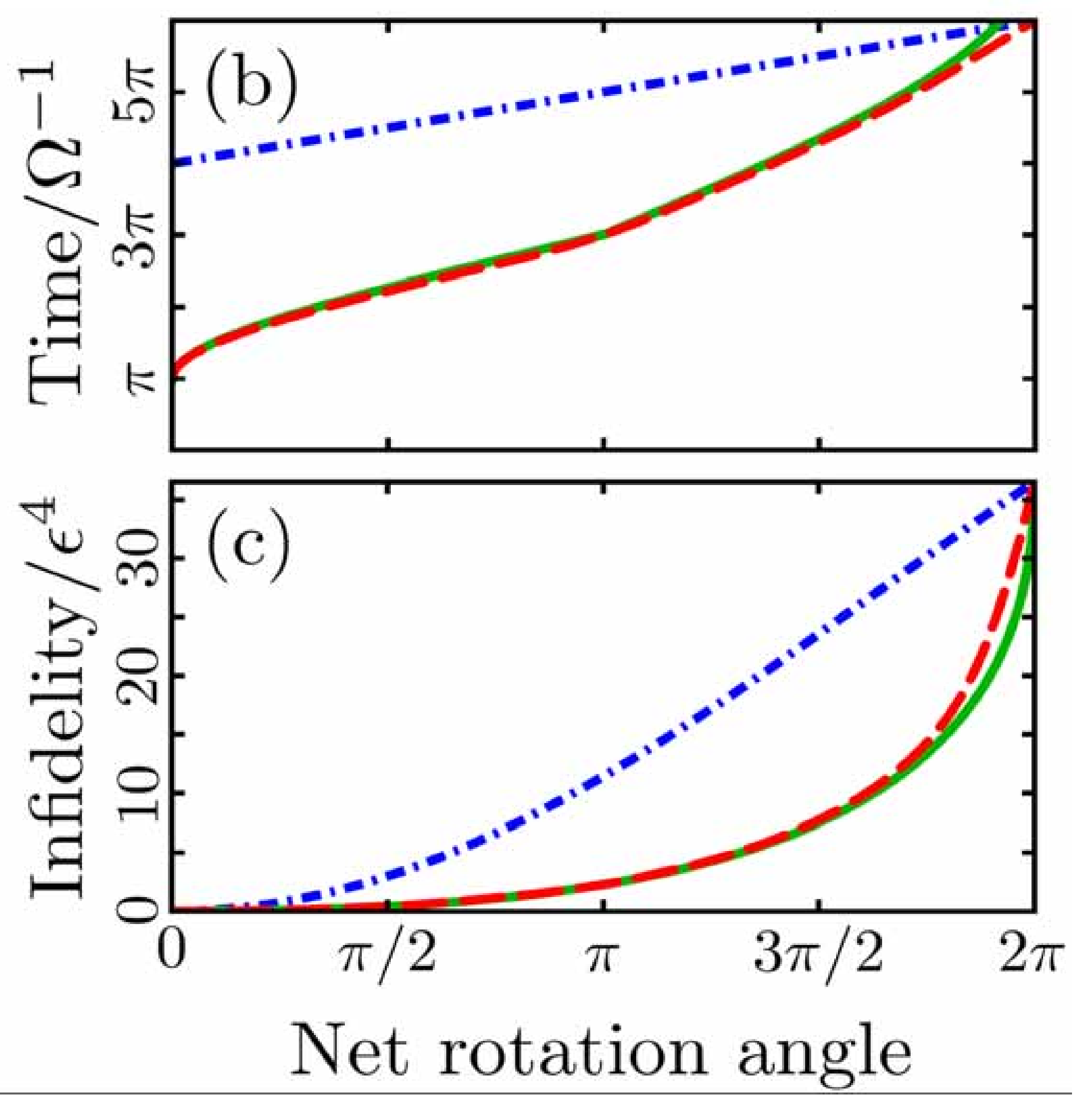
Transformed to the plane



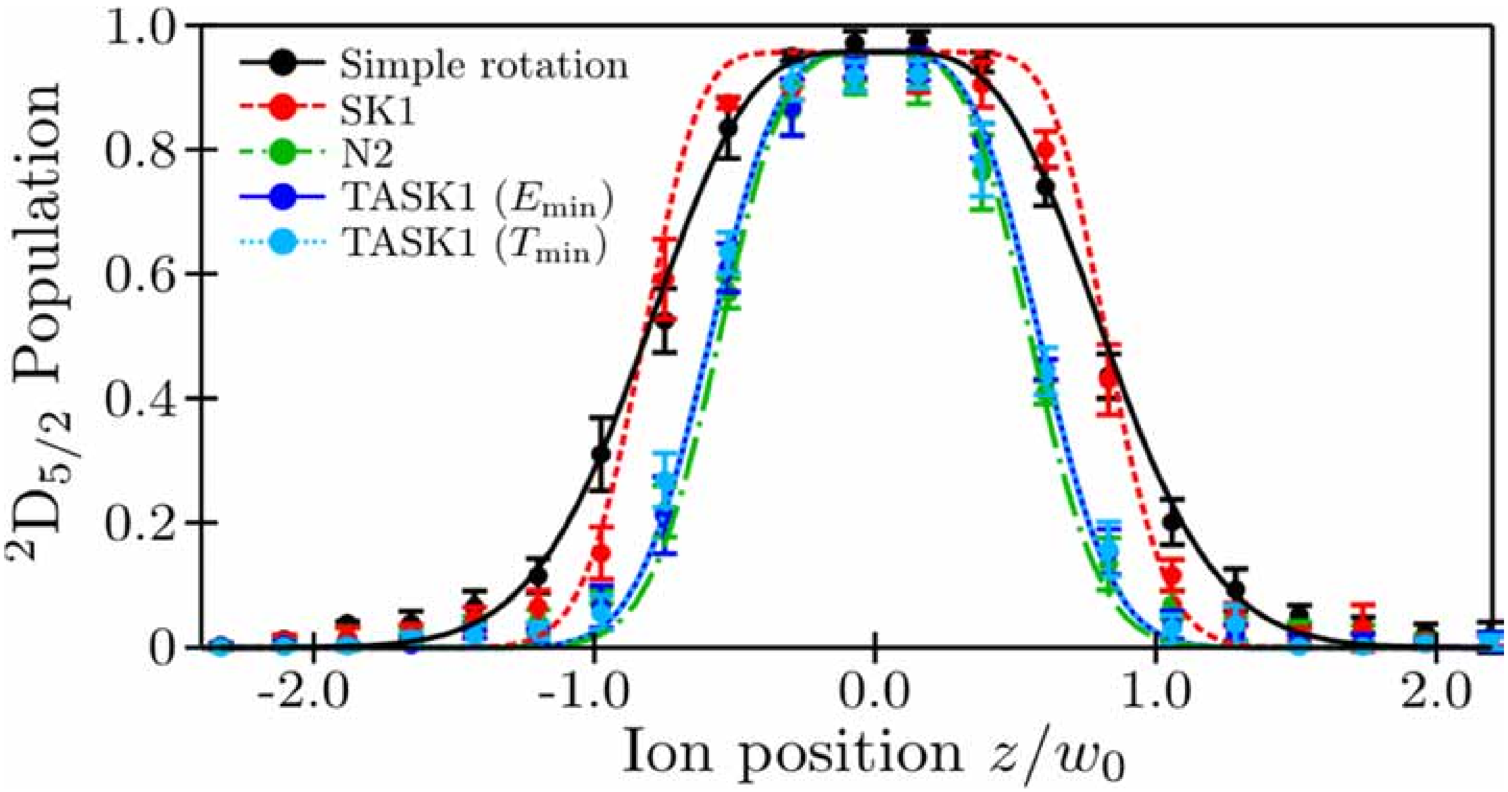
Optimal solutions



Fast is also low error



Pulse sequence ion addressing



Move ion through stationary laser.

Merrill, Vittorini, Addison, KRB, and Doret, Phys. Rev. A(R), 2014

Conclusions

- Quantum control can improve fidelity when the errors are
  - Coherent
  - Weak
  - Slowly fluctuating
- Quantum control can also reduce spatial and temporal correlations in the error
- Despite 50 years of history, protocols are still improving though both better theoretical ideas and improved numerical methods
  - Two-qubit gates still need help