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A General Transfer-Function Approach to Noise Filtering in Open-Loop Quantum Control

Lorenza Viola

Dept. Physics & Astronomy
Dartmouth College

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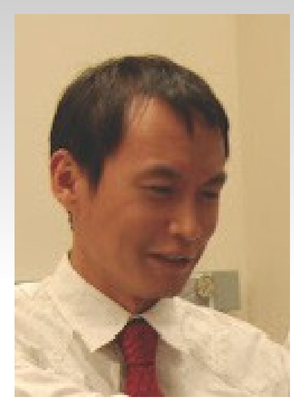
Gerardo Paz-Silva
Dartmouth

Michael Biercuk
& Todd Green
U. Sydney

Ken Brown &
Chingiz Kabytaev
GeorgiaTech

Will Oliver
MIT



Motivation

Goal: High-precision, robust control of *realistic* quantum-dynamical systems.

- Real-world quantum control systems typically entail:
  - ➞ *Noisy*, irreversible open-system dynamics…
  - ➞ *Imperfectly characterized* dynamical models…
  - ➞ *Limited* control resources…
  - ⋮

- Broad significance across coherent quantum sciences:
  - ➞ High-resolution imaging and spectroscopy…
  - ➞ Quantum chemistry and biology…
  - ➞ Quantum metrology, sensing and identification…
  - ➞ High-fidelity QIP, fault-tolerant QEC…
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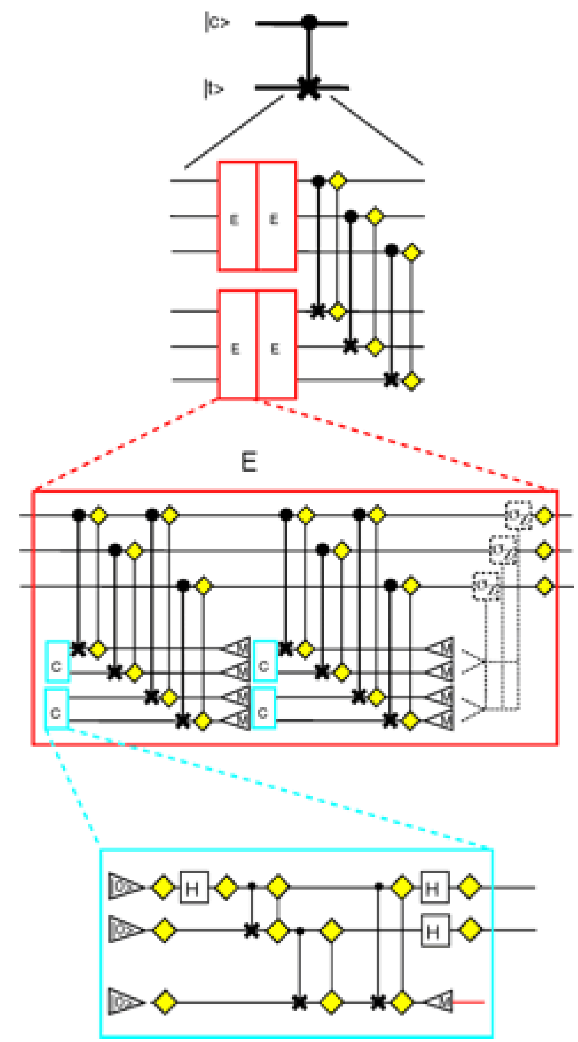

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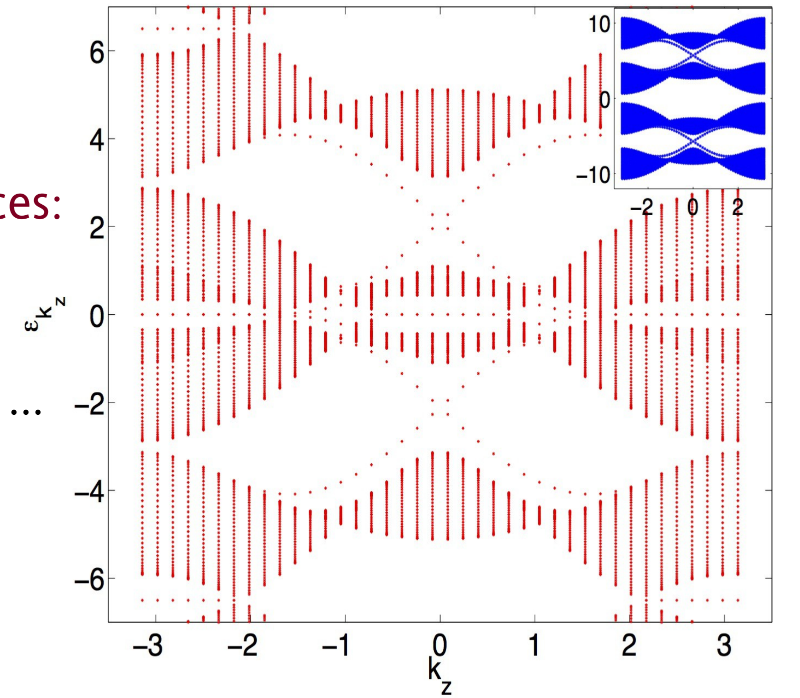
Poudel, Ortiz & LV,
Floquet Majorana flat bands,
ArXiv:1412.2639



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  - ➞ High-fidelity QIP, fault-tolerant QEC...
  - ➞ Engineering of novel quantum matter...
  - ⋮

The premise: Dynamical QEC

Open-loop *Hamiltonian engineering* [both *closed* and *open* systems]:
Dynamical control solely based on *unitary* control resources.

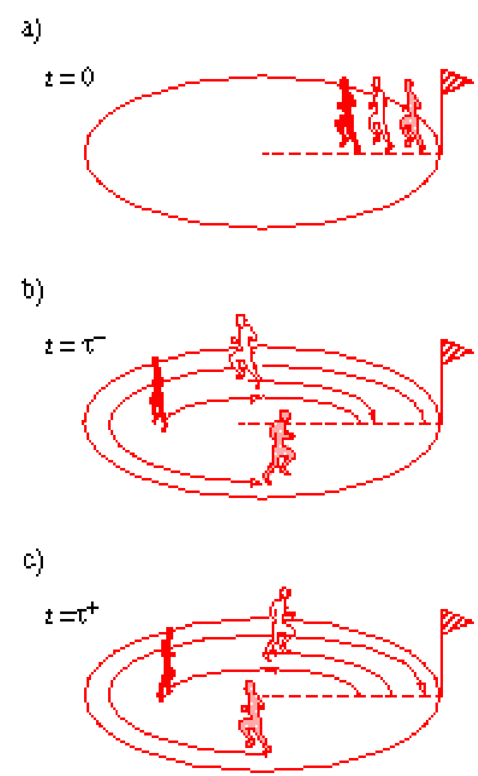
Simplest setting: Multi-pulse decoherence control for quantum memory ⇒ DD

LV & Lloyd, PRA 1998.

Key principle: Time-scale separation ⇒ 'Coherent averaging'

Paradigmatic example: Spin echo ⇔ Effective time-reversal

Hahn, PR 1950.



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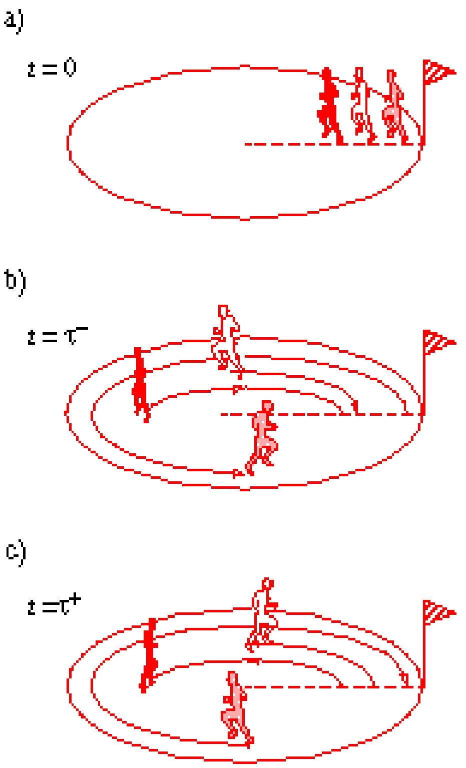


Key features: 'Non-Markovian' quantum dynamics

(1) Dynamical error suppression is achieved in a *perturbative* sense

$$\frac{\tau_{\mathrm{ctrl}}}{\tau_{\mathrm{corr}}} \sim \omega_c \tau_{\mathrm{ctrl}} \quad \textit{small} \text{ parameter}$$

(2) Unwanted dynamics may include coupling to *quantum* bath

(3) Dynamical QEC is achievable *without* requiring full/quantitative knowledge of error sources
[⇒ built-in robustness against 'model uncertainty']

Quantum control tasks

- Hamiltonian engineering techniques provide a versatile tool for *dynamical control* and *physical-layer* decoherence suppression in a variety of QIP settings:

  ➞ Arbitrary state preservation ⇒ DQEC for *quantum memory*
  - ✔ Pulsed DD – 'Bang-Bang' (BB) limit/instantaneous pulses
  - ✔ Pulsed DD – Bounded control ('Eulerian')/'fat' pulses
  - ✔ Continuous-(Wave, CW) [always-on] DD

  ➞ Quantum gate synthesis ⇒ DQEC for *quantum computation*
  - ✔ Hybrid DD-QC schemes – BB, w or w/o encoding
  - ✔ Dynamically corrected gates (DCGs) – Bounded control only
  - ✔ Composite pulses – Bounded control only

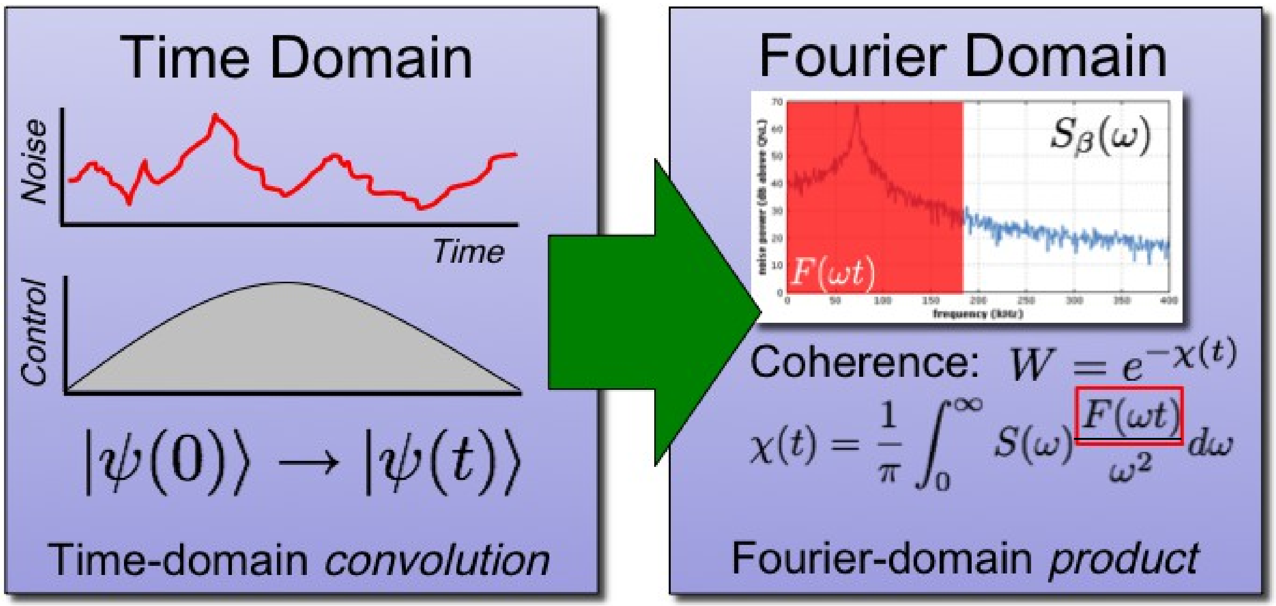
  ➞ Quantum system identification ⇒ Dynamical control for *signal/noise estimation*
  - ✔ Signal reconstruction – dynamic parameter estimation ('Walsh spectroscopy')
  - ✔ Spectral reconstruction – DD noise spectroscopy

  ➞ Hamiltonian synthesis ⇒ Dynamical control for *quantum simulation*
  - ✔ Closed-system [many-body, BB and Eulerian] Hamiltonian simulation
  - ✔ Open-system [dynamically corrected] Hamiltonian simulation

  ⋮

Time vs frequency domain: Filter transfer functions

Kurizki *et al* PRL 2001; Uhrig PRL 2007; Cywinski *et al*, PRB 2008; Khodjasteh *et al*, PRA 2011; Biercuk *et al*, JPB 2011; Hayes *et al*, PRA 2011; Green *et al*, PRL 2012, NJP 2013; Kabytayev *et al*, PRA 2014...



FILTER FUNCTION (FF)

$$F(\omega t) \equiv \omega^2 |\mathcal{F}[\,y(t)]|^2$$

- Picture the control modulation as enacting a 'noise filter' in frequency domain:
  - ➞ Simplest case: Single qubit under *classical Gaussian dephasing*, DD via perfect π pulses

$$U_{\rm ctrl}(t) = \sigma_x^{(y(t)+1)/2} = \begin{cases} \mathbb{I} \\ \sigma_x \end{cases} \qquad F(\omega t) \propto (\omega t)^{2(\delta+1)}, \;\; \omega \rightarrow 0$$

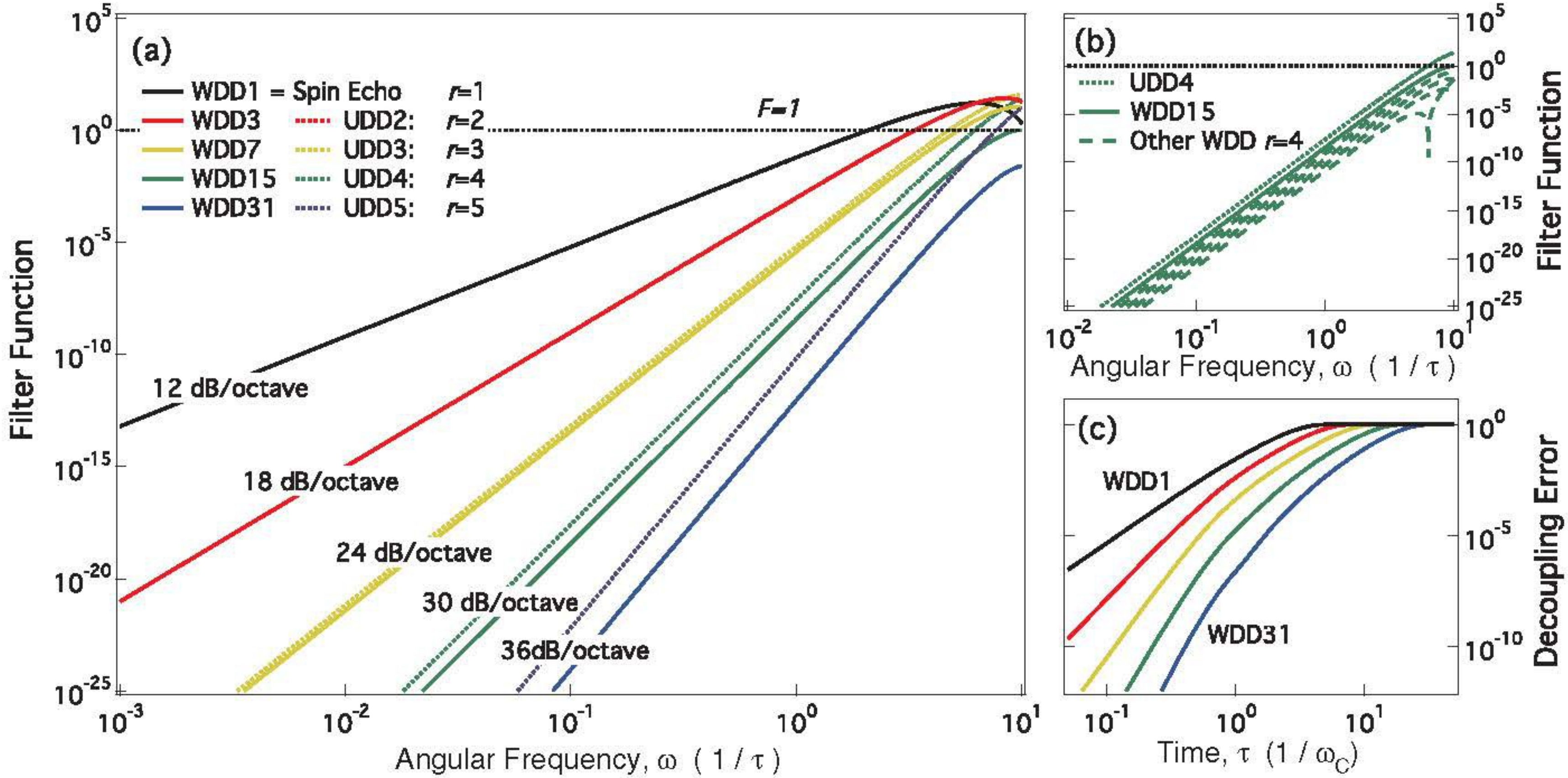
  - ➞ The larger the *order of error suppression* δ, the higher the degree of noise cancellation:

$$\rho(t) \approx \rho(0) + \mathcal{O}(t^{\delta+1})$$

Filter transfer function approach: Advantages...

Hayes, Khodjasteh, LV & Biercuk, PRA 84 (2011).

HIGH-PASS NOISE FILTERING



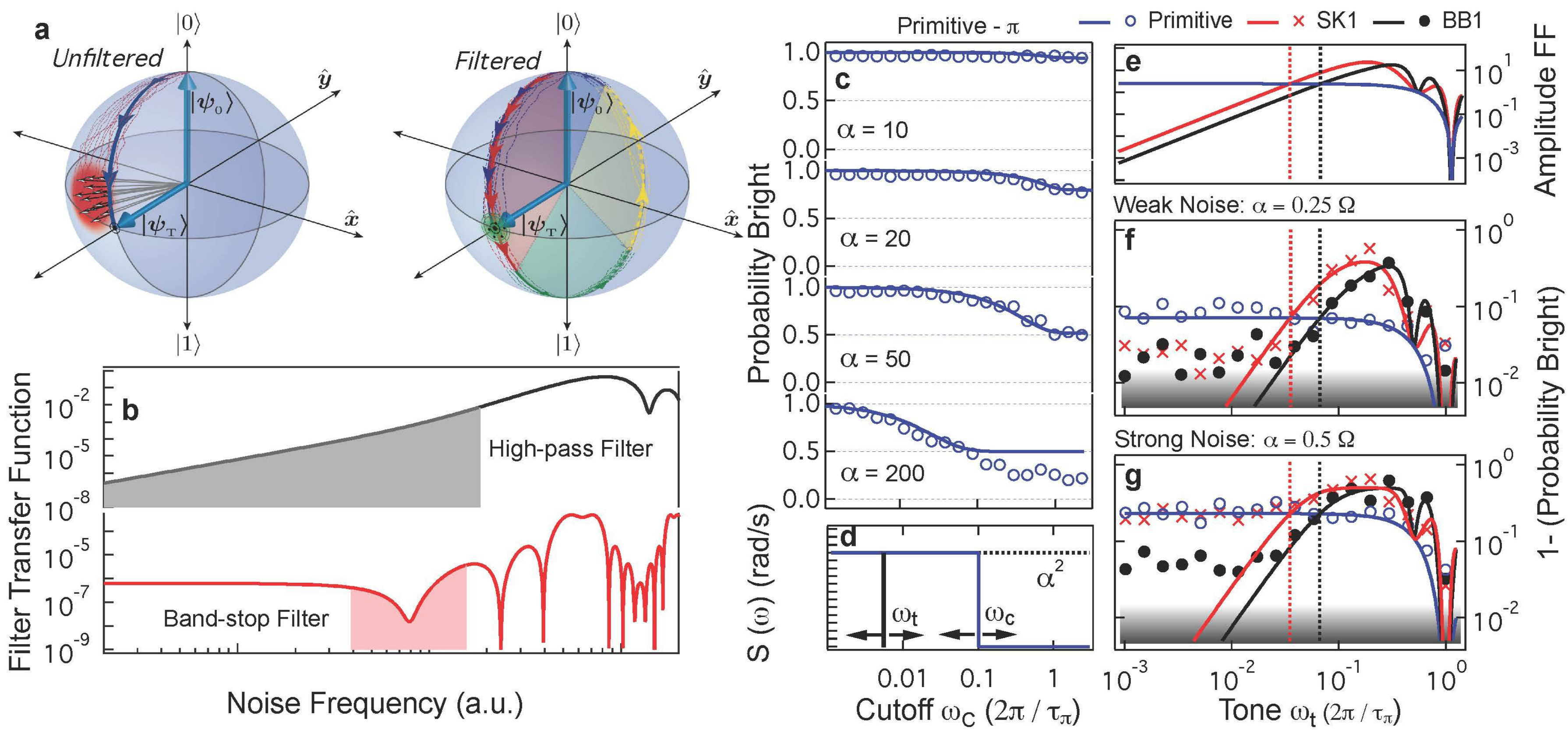
➞ Direct contact with signal processing, [classical and quantum] *control engineering*...

➞ Simple *analytical* evaluation of control performance, compared to numerical simulation...

➞ Natural starting point for analysis and synthesis of control protocols *tailored to specific spectral features of generic time-dependent noise*...

Filter transfer function approach: Validation...

Soare *et al*, Nature Phys. (Oct 2014).



- ➞ Control objective: noise-suppressed single-qubit π rotations under [non-Markovian] amplitude control noise ⇒ Generalized FF formalism.

  Green *et al*, PRL 2012, NJP 2013.

- ➞ Control protocols: [NMR] composite-pulse sequences.
- ➞ Quantitative agreement with analytical FF predictions observed in the weak-noise limit.

Filter transfer function approach: Assessment...

- Major limitation of current generalized FF (GFF) formalism:
  High-order GFFs are given in terms of an infinite recursive hierarchy – awkward!
  - ➡ Explicit calculations to date ⇒ Single-qubit controlled dynamics under *classical* noise: *lowest-order* fidelity estimates, *Gaussian* [stationary] noise statistics...

  ...
  - ➡ *Higher-order* terms *are* [already] of relevance to quantum control experiments...
  - ➡ What about *general [quantum and/or non-Gaussian] noise models*?...
  - ➡ What about *general target [multi-qubit] systems*?...

Filter transfer function approach: Next steps...

- Major limitation of current generalized FF (GFF) formalism:
  High-order GFFs are given in terms of an infinite recursive hierarchy – awkward!
  - ➞ Explicit calculations to date ⇒ Single-qubit controlled dynamics under *classical* noise: *lowest-orde*r fidelity estimates, *Gaussian* [stationary] noise statistics...

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  - ➞ What about *general target [multi-qubit] systems*?...

- Assuming that a *general* frequency-domain description is viable, to what extent will it be *equivalent* to the time-domain description...
  - ➞ How to rigorously characterize the *filtering capabilities* of a control protocol?...

<u>Challenge:</u>
To build a general theory for *open-loop noise filtering* in non-Markovian quantum systems.

Control-theoretic setting: System and noise

- Target system S (finite-dim) coupled to *quantum or classical* environment [bath] B:

$$H(t) = H_S \otimes \mathbb{I}_B + H_{SB}(t)$$

  with respect to interaction picture defined by $\mathbb{I}_S \otimes H_B$.

  ➞ Classical noise formally recovered for $H(t) \equiv H_S(t)$ [stochastic time-dependence]

- Environment B is uncontrollable $\Rightarrow$ Controller acts directly on S *alone*:

$$H_{\text{tot}} = H(t) + H_{\text{ctrl}}(t) \otimes \mathbb{I}_B \equiv \underbrace{H_0(t)}_{\text{error-free}} + \underbrace{H_e(t)}_{\text{unwanted}}$$

$$H_e(t) = H_{SB}(t) + H_{\text{ctrl},e}(t) \otimes \mathbb{I}_B + H_{S,e} \otimes \mathbb{I}_B$$

  ➞ Evolution under ideal Hamiltonian over time T yields the desired unitary gate Q on S (e.g., $Q = \mathbb{I}_S$ for DD).

Control-theoretic setting: Isolating the noise

- Total [joint] propagator may be *exactly* expressed in terms of 'error propagator':

$$U(T) = U_0(T)\tilde{U}_e(T) \equiv Q\,\tilde{U}_e(T) \qquad \tilde{U}_e(t) = \mathcal{T}\exp\left\{-\,i\int_0^t \tilde{H}_e(s)ds\right\}$$

➞ Choose an Hermitian operator basis on S, $\{\mathbb{I}_S, O_v\}$, $\mathrm{Tr}[O_v]=0 \;\Rightarrow$

$$\boxed{\tilde{H}_e(t) = U_0(t)^\dagger H_e(t) U_0(t) \equiv \sum_{u,v} y_{uv}(t)\, O_v \otimes B_u(t)}$$

target-dependent control matrix

- Error propagator may be formally computed via a Magnus series expansion:

$$\tilde{U}_e(T) = e^{\sum_{\alpha=1}^{\infty}\Omega_\alpha(T)} \equiv e^{-iT(H^{\mathrm{eff}}_{SB}(T)+H^{\mathrm{eff}}_{B}(T))} \equiv e^{-i\Omega_e(T)}$$

➞ α-th order Magnus term $\Omega_\alpha(T)$ involves α-th order nested commutators of $\tilde{H}_e(t_j)$.

Cancellation order in time domain

- Magnus series has traditionally been used to characterize error-suppression properties of a control protocol in the *time domain*:

$$\tilde{U}_e(T) = e^{\sum_{\alpha=1}^{\infty} \Omega_\alpha(T)} \equiv e^{-iT(H_{SB}^{\text{eff}}(T)+H_B^{\text{eff}}(T))} \equiv e^{-i\Omega_e(T)}$$

<u>Strategy:</u> [perturbatively] minimize the sensitivity of the controlled evolution to $H_e(t)$, by making $\tilde{U}_e(T)$ *as close as possible* to a 'pure-bath' evolution [identity on S...]

Khodjasteh, Lidar & LV, PRL 2010; Khodjasteh, Bluhm & LV, PRA 2012.

- <u>Definition.</u> A control protocol specified by $y_{uv}(t)$ achieves cancellation order (CO) δ if the norm of the error action operator $\Omega_e(T)$ [up to pure-bath terms] is reduced, such that the leading-order correction mixing S and B scales as

$$\boxed{||TH_{SB}^{\text{eff}}(T)|| = \mathcal{O}(T^{\delta+1})}$$

➞ CO = Standard 'decoupling order' for a DD protocol (e.g., CDD, WDD, UDD...)

Generalized filter functions-1

- GFFs may be most generally defined directly *at the level of the effective Hamiltonian*:
  - ➡ Express each $\tilde{H}_e(t_j)$ in the α-th order term wrto the chosen operator basis:

$$\Omega_\alpha(T) = \sum_{\vec{u},\vec{v}} \int_{\{0\leq t_\alpha \ldots \leq t_2 \leq t_1 \leq T\}} f(\{y_{[\alpha]}\})\, O_{v_1} \cdots O_{v_\alpha} \otimes B_{u_1}(t_1) \cdots B_{u_\alpha}(t_\alpha)\, d^\alpha \vec{t}$$

  - ➡ Express each bath variable in terms of corresponding *frequency-Fourier transform*:

$$B_u(t) \equiv \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, e^{i\omega t} B_u(\omega)$$

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$$\boxed{\Omega_\alpha(T) = -i \sum_{\vec{u},\vec{v}} \int \frac{d^\alpha \vec{\omega}}{(2\pi)^\alpha}\, G^{(\alpha)}_{\vec{u}\vec{v}}(\vec{\omega}, T)\, O_{v_1} \cdots O_{v_\alpha} \otimes B_{u_1}(\omega_1) \cdots B_{u_\alpha}(\omega_\alpha)}$$

Meaning: α-th order GFF describes the *filtering effect* of the applied control on the corresponding 'operator string' in the α-th order Magnus term.

Generalized filter functions-2

- GFFs naturally appear in the *reduced* (or *ensemble-averaged*) *system dynamics*:
  - ➞ Work in a basis where Q is diagonal and assume initial S-B factorization:

$$\rho_{\ell\ell'}(T) = q_\ell^* q_{\ell'} \sum_{m,m'} \rho_{m,m'}(0)\, \mathrm{Tr}_B[\langle \ell|\tilde{U}_e(T)|m\rangle \rho_B \langle m'|\tilde{U}_e(T)^\dagger|\ell'\rangle]$$

  - ➞ By Taylor-expanding $\tilde{U}_e(T) = \sum_{r=0}^{\infty}(-i\,\Omega_e(T))^r/r!$, and using the definition of GFFs, a *common structure* may be identified in each contributing term:

$$\langle \ell|O_{[\alpha_1]}\cdots O_{[\alpha_r]}|m\rangle\langle m'|O_{[\alpha'_1]}\cdots O_{[\alpha'_{r'}]}|\ell'\rangle$$

$$\int \mathcal{D}\vec{\omega}\; \underbrace{G^{(\alpha_1)}\ldots G^{(\alpha_r)}G^{*(\alpha'_1)}\ldots G^{*(\alpha'_{r'})}}_{\boxed{\text{filtering properties}}}\; \underbrace{\mathrm{Tr}_B[\rho_B B_{[\alpha_1]}\cdots B_{[\alpha_r]}B_{[\alpha'_1]}\cdots B_{[\alpha'_{r'}]}]}_{\boxed{\text{noise properties}}}$$

$\Rightarrow$ related to high-order noise power spectra

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$\Rightarrow$ related to high-order noise power spectra

Example:

BB DD of a single-qubit under *Gaussian*, stationary dephasing noise [again!]

$$\chi(T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G^{(1)}(\omega, T) G^{(1)}(-\omega, T) S(\omega)\, d\omega\,, \quad G^{(1)}(\omega, T) = \int_0^T dt\, y(t) e^{i\omega t}$$

$$F(\omega t) \equiv \omega^2 G^{(1)}(\omega, t) G^{(1)}(-\omega, t)$$

Fundamental filter functions

- Key insight: GFFs share a *common structure*, determined by [infinite in general, but] easily computable set of 'elemental' FFs ⇒ *fundamental filter functions* (FFFs):

$$F^{(\alpha)}_{\vec{u}\vec{v}}(\vec{\omega}, T) \equiv (-i)^\alpha \int_{\{0\leq t_\alpha \leq \ldots t_1 \leq T\}} d^\alpha \vec{t} \prod_{j=1}^{\alpha} \left(y_{u_j v_j}(t_j) e^{i\omega_j t_j} \right)$$

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- Theorem: Arbitrary GFFs of order α, $\alpha = 1, \dots, \infty$, may be exactly represented as

$$-iG^{(\alpha)}_{\vec{u}\vec{v}}(\vec{\omega},T) = F^{(\alpha)}_{\vec{u}\vec{v}}(\vec{\omega},T) - \sum_{j=2}^{\alpha} \frac{(-1)^j}{j} \sum_{\sum_{r=1}^{j} \alpha_r = \alpha} \prod_{k=1}^{j} F^{(\alpha_k)}_{\vec{u}_{[s_{k-1},s_k]}\vec{v}_{[s_{k-1},s_k]}}(\vec{\omega}_{[s_{k-1},s_k]},T)$$

➞ Proof follows from exact relationship between Magnus and Dyson series expansion.

Key point: Arbitrary high-order GFFs are *explicitly, non-recursively* computable as combinations of FFFs of same and lower order.

Filtering order in frequency domain

Question: To what extent do FFFs characterize filtering properties of a protocol?

- Complete information about filtering behavior is encoded in principle in the set of all 'relevant' GFFs – $O_{v_1}\cdots O_{v_\alpha} \neq \mathbb{I}_S$ in *at least one* factor [no pure-bath evolution].

  ➡ For each GFF [FFF], define generalized [fundamental] CO and filtering order (FO) as

$$G^{(\alpha)}_{\vec{u}\vec{v}}(\vec{\omega},T) \sim \mathcal{O}(m^{\Phi^{(\alpha)}_{\vec{u}\vec{v}}}(\vec{\omega}-\vec{\omega}_0)\,T^{\Delta^{(\alpha)}_{\vec{u}\vec{v}}+1})$$
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- Definition. For a control protocol specified by $y_{uv}(t)$, the generalized and fundamental cancellation order Δ and δ are given by the *minimum over all the relevant GFFs/FFFs:*

$$\Delta = \min_{\mathcal{R}_\alpha,\forall\alpha}\{\Delta^{(\alpha)}_{\vec{u}\vec{v}}\}, \quad \delta = \min_{\mathcal{R}_\alpha,\forall\alpha}\{\delta^{(\alpha)}_{\vec{u}\vec{v}}\}$$

  The generalized and fundamental filtering order Φ and ϕ at level κ are given by the *minimum over all the relevant GFFs/FFFs up to Magnus order κ:*

$$\Phi^{[\kappa]} = \min_{\mathcal{R}_\alpha,\alpha\leq\kappa}\{\Phi^{(\alpha)}_{\vec{u}\vec{v}}\}, \quad \phi^{[\kappa]} = \min_{\mathcal{R}_\alpha,\alpha\leq\kappa}\{\phi^{(\alpha)}_{\vec{u}\vec{v}}\}$$

Filtering vs. cancellation order

- Theorem: The generalized and fundamental FO and CO are related in general as follows:

$$\Phi^{[\kappa]} = \phi^{[\kappa]}, \quad \kappa = 1, \ldots, \infty; \qquad \Delta = \delta$$
$$\phi^{[\infty]} \leq \delta$$

Key point 1: Access to FFFs suffices to *fully* characterize the CO and FO that protocol can guarantee *under minimal assumptions on the noise model.*

➡ Higher effective CO and FO are possible *given specific knowledge* on the noise model.

➡ Level-κ FOs are *not* a priori constrained, and the inequality at $\kappa = \infty$ *can be strict.*

Key point 2: Cancellation and filtering are in general two *inequivalent notions.*

Case study: Dynamical decoupling

- Simplest setting: Single-axis control protocols $\Rightarrow$
  Ideal, single-qubit DD in the presence of arbitrary, *non-Gaussian* dephasing

  Claim: Arbitrarily high-order filtering may be achieved for ideal single-axis DD via concatenation, CO $= \delta = \phi^{[\infty]} =$ FO for CDD$_\delta$.

  ➞ This feature is *not generic* to high-order DD protocols! E.g. δ-th order Uhrig DD:

  CO $= \delta$, FO $= \phi^{[\infty]} \leq 1$ or 2 for UDD$_\delta$, $\delta \leq 8$.

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  $$\text{CO} = \delta, \quad \text{FO} = \phi^{[\infty]} \leq 1 \text{ or } 2 \text{ for UDD}_\delta,\ \delta \leq 8.$$

  ➞ Illustrative toy models:

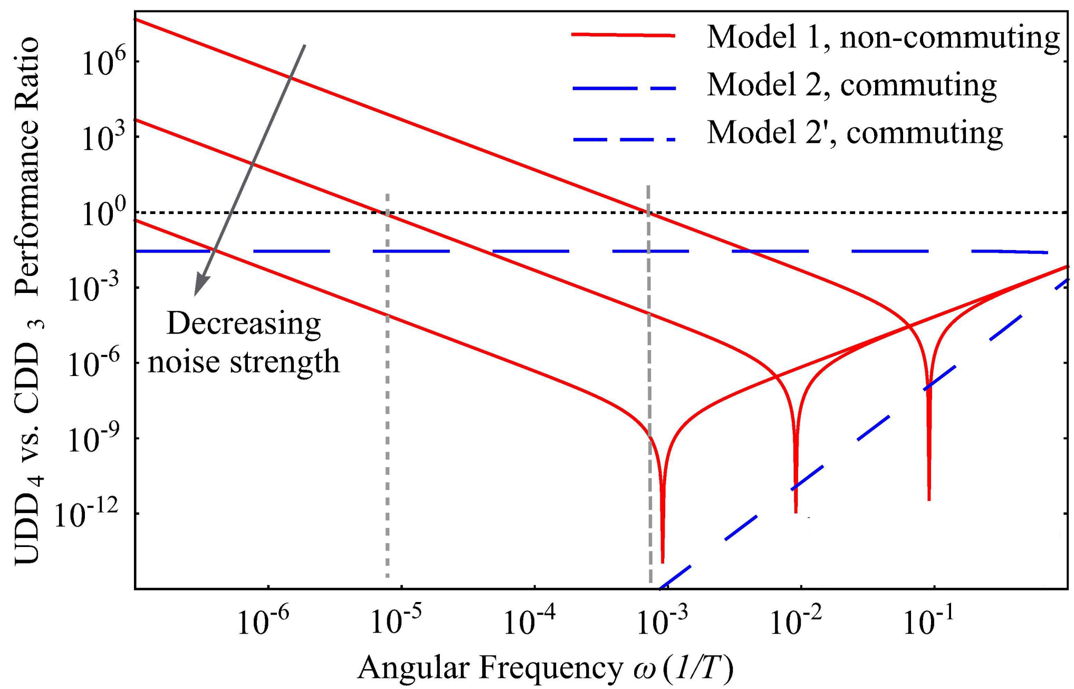
  $$\tilde{H}_{e,1}(t) = gy(t)\sigma_z \otimes [B_z(t) + B_y(t)]$$

  $$\tilde{H}_{e,2}(t) = gy(t)\sigma_z \otimes B_z(t)$$

  CDD$_3$: CO $= 3$, FO $= 3$

  UDD$_4$: CO $= 4$, FO $= 2$

  Inversion of performance at low frequencies, due to high-order Magnus terms



Further examples

- General case: Multi-axis control protocols
  E.g., DD with imperfect/bounded control, DCGs, composite pulses...

  Claim: A protocol which does not achieve perfect cancellation of arbitrary quasi-static noise has vanishing FO, $\phi^{[\infty]} = 0$.

Meaning: Arbitrarily high-order filtering is too strong a requirement – *finite-κ filtering is relevant in practice.*

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Soare *et al*, Nature Phys. (Oct 2014).

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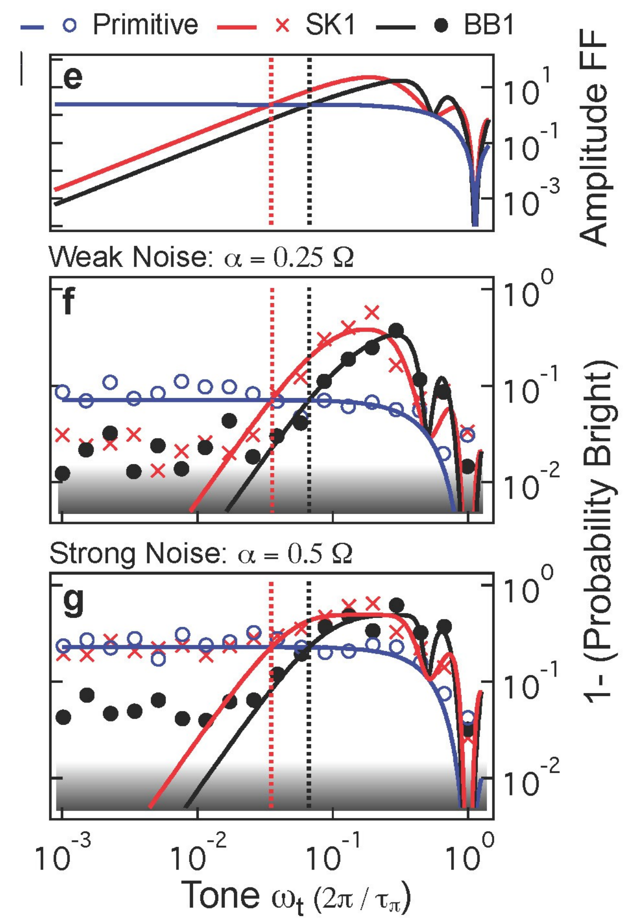
➞ Illustrative example: NMR composite-pulse sequences

$$\text{SK1: CO} = 1, \text{FO} = 1$$
$$\text{BB1: CO} = 2, \text{FO} = 1$$

➞ Distinction between CO and FO is *relevant to current quantum-control experiments* and [already] informing novel approaches to control synthesis…



Conclusion and outlook

- A general, *computationally tractable* approach to open-loop noise filtering in [non-Markovian] open quantum systems is possible based on identifying a set of *fundamental FFs* – out of which *arbitrary* generalized FFs may be directly assembled.
- Fundamental FFs *suffice to characterize the error-suppression capabilities in both the time and frequecy domain* under minimal assumptions on the noise model.
- Order of error cancellation [a-la-Magnus] and order of filtering are in general t*wo inequivalent and potentially equally relevant notions for time-dependent noise.*

Conclusion and outlook

- A general, *computationally tractable* approach to open-loop noise filtering in [non-Markovian] open quantum systems is possible based on identifying a set of *fundamental FFs* – out of which *arbitrary* generalized FFs may be directly assembled.

- Fundamental FFs *suffice to characterize the error-suppression capabilities in both the time and frequency domain* under minimal assumptions on the noise model.

- Order of error cancellation [a-la-Magnus] and order of filtering are in general t*wo inequivalent and potentially equally relevant notions for time-dependent noise.*

- Additional investigation is needed to appreciate the *full theoretical and experimental significance* of filtering perspective for open-loop quantum control:

  ➞ Multi-qubit DD/long-time quantum-memory settings;

  Paz-Silva, S.-W. Lee, T. J. Green & LV, forthcoming.

  ➞ Analytical and/or numerical synthesis of 'customized' noise filters;

  ➞ Protocols for *non-Gaussian* noise identification/sensing;

  Paz-Silva, L. Norris & LV, forthcoming.

  ➞ Implications for [non-Markovian] quantum fault tolerance?...

  ⋮