Can long-range interactions stabilize a quantum memory at non-zero temperature?

Olivier Landon-Cardinal
olc@caltech.edu

INSTITUTE FOR QUANTUM INFORMATION AND MATTER

Institute for Quantum Information and Matter (IQIM)
California Institute of Technology (Caltech)

CALIFORNIA INSTITUTE OF TECHNOLOGY 1891

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Self-correcting memory

Self-correcting memory = physical system which encode (quantum) information
- reliably
- for a macroscopic period of time
- letting the memory interact with its environment (thermal noise)
- *without* active error correction

Encoding $|\psi_i\rangle \xrightarrow{\text{Noise}} |\psi_i'\rangle \xrightarrow{\text{Decoding}} |\psi_f\rangle$

Code = subspace of dim. >1 whichs encodes the quantum information.

Typically, the degenerate groundspace of a local Hamiltonian of spin particles (qudits) on a 2D/3D lattice.

Quantum  hard drive?

Long-range interactions in quantum memory

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olc@caltech.edu

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Desiderata for quantum memory
- Perturbations
- Thermal stab.

Effective long-range interactions
- Chemical pot.
- Stability?

Perturbative analysis
- Analytics
- Q. field theory
- Symm. protection?

Self-correcting quantum memory

Quantum system with a degenerate groundspace.

Desiderata

(i) Thermal stability

Memory time grows unbounded as system size L gets larger.

$$\tau_{\mathrm{mem}} \sim e^{L} \quad \text{or} \quad (\sim \mathrm{poly}(L))$$

➡ must be true for exact Hamiltonian but also under local perturbation

(ii) Robustness to local perturbations

Degeneracy of the ground space cannot be lifted by local perturbation.

Thermal stability for exact H + robustness to local perturbations
is sufficient for gapped system.

Robustness to local perturbations for gapped system?

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Robustness to local perturbations

Local commuting projector code (LCPC)

N d-dim. spin particles (qudits) located on the vertices of a lattice Λ

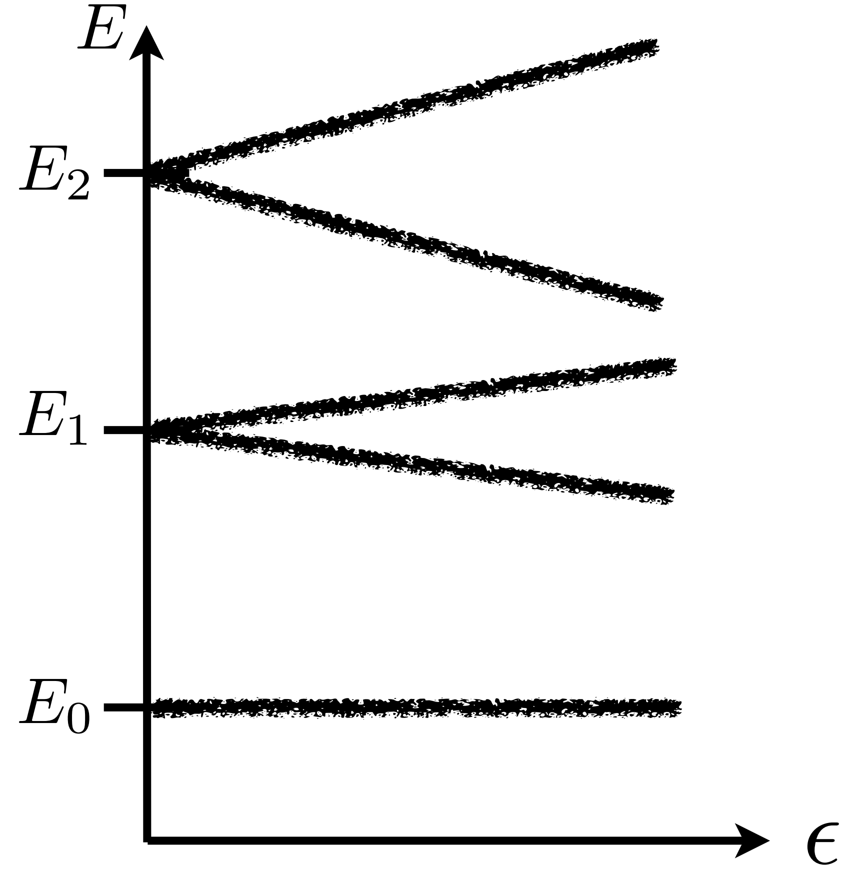
$$H = -\sum_{X \subset \Lambda} P_X$$

- projectors $(P_X)^2 = P_X$
- terms commute $[P_X, P_Y] = 0$
- local $\mathrm{diam}(X) \geq w \Rightarrow P_X = 0$
- frustration-free $\forall X \quad P_X|\Omega\rangle = +|\Omega\rangle$

Spectrum of LCPC Hamiltonian is **stable** if the Hamiltonian is topologically ordered.

Bravyi, Hastings, Michalakis. J. Math. Phys. **51** 093512 (2010)

Local perturbation $H \to H + \epsilon \sum_{\substack{X \subset \Lambda \\ \mathrm{diam}(X) < w}} V_X$



Remark: the op. norm of the perturbation grows with system size. $\|V\| = L^D \|V_X\|$

➡No degeneracy lifting

➡Gap does not close

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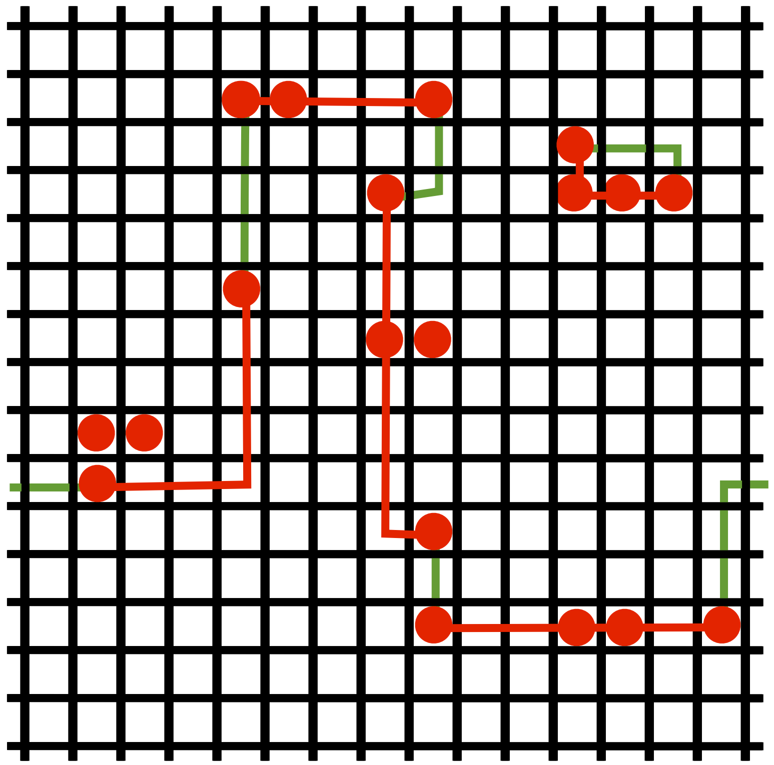
2D topological systems are thermally unstable

2D toric code A. Y. Kitaev, Ann. Phys. 303, 2 (2003).



Point-like excitations = anyons

End points of string logical operator

$$[H, T] = 0$$

Non-zero temperature: finite density of anyons

➡ (1) constant energy cost to create pair of anyons

Anyons propagate at no energy cost (thermal deplacement)

➡ (2) no energy cost to propagate anyons

Memory time is a constant, independent of system size.

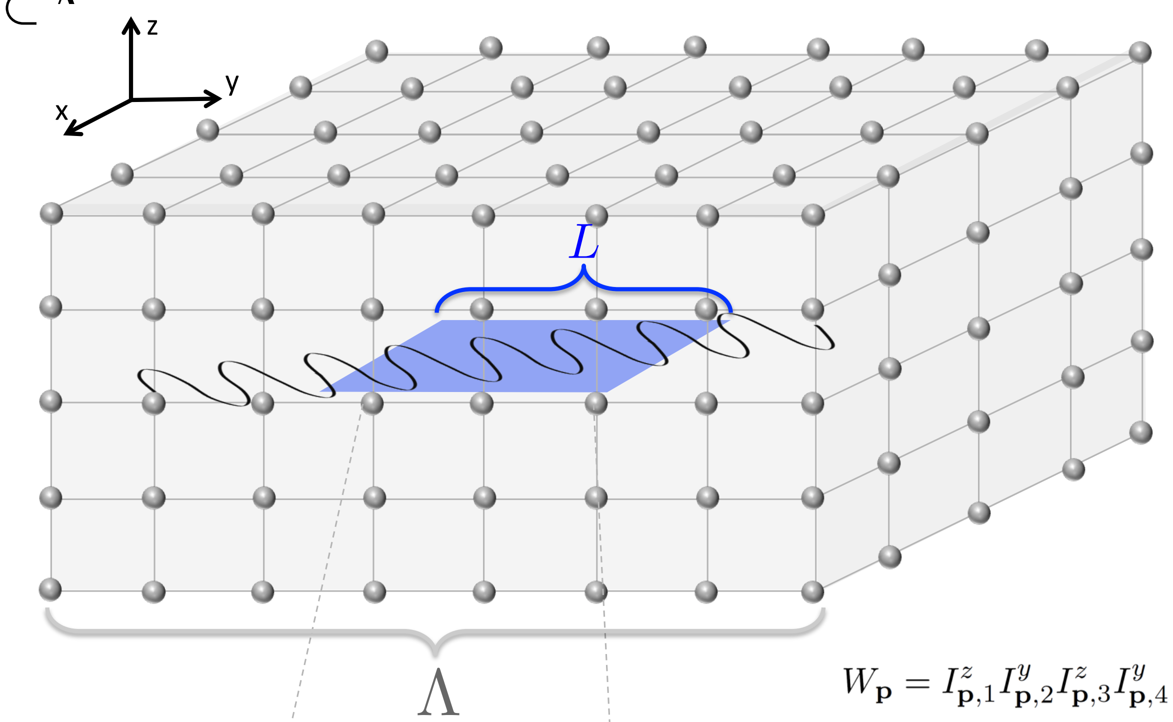
R. Alicki, M. Fannes, and M. Horodecki, J. Phys. A: Math. Gen. **42**, 065303 (2009).

How to avoid points (1) and (2)?

Effective long-range interactions between anyons

Couple the topological system to an auxiliary bath

$$H = -\sum_{X\subset\Lambda} P_X \otimes \mathbb{I}_{\mathcal{A}} + \sum_{X\subset\Lambda} g_X P_X \otimes \Theta_X + \mathbb{I}_{\mathcal{S}} \otimes H_{\mathcal{A}}$$



Toric-boson model

Hamma, Castelnovo, Chamon.
PRB **79**, 245122 (2009)

Repulsive interaction

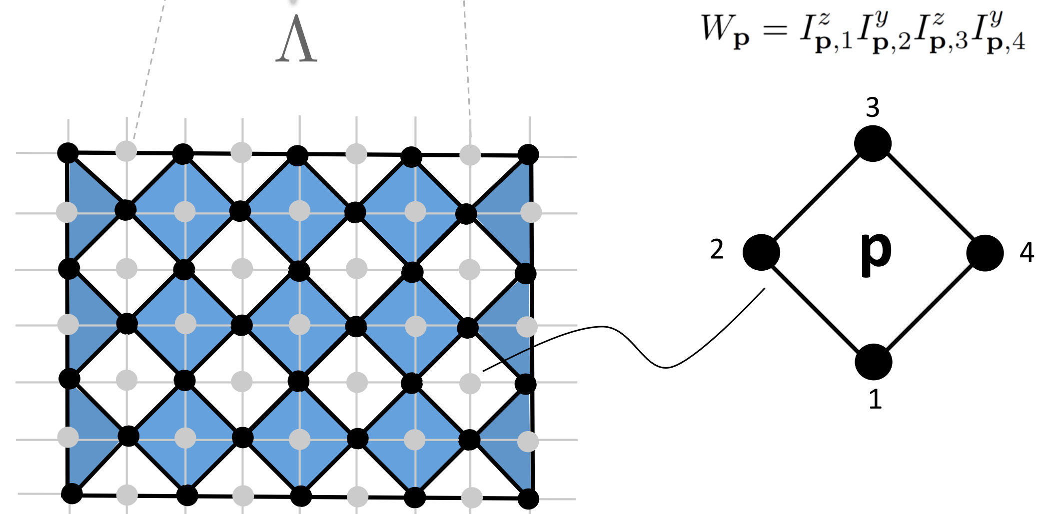
Chesi, Röthlisberger, Loss. PRA **82**, 022305 (2010)

Coupling to free bosons

Pedrocchi, Hutter, Wootton, Loss.
PRA **88**, 062313 (2013)

$$H = H_b + A\sum_p W_p \otimes (a_p + a_p^\dagger)$$

$$H_b = \epsilon_0 \sum_i a_i^\dagger a_i - t\sum_{\langle i,j\rangle} a_i^\dagger a_j$$



$$H = -\sum_{p,p'} J_{p,p'} W_p W'_p + H'_b$$

$$J_{p,p'} = \frac{A^2}{|r_p - r_{p'}|}$$

2 effects

1. Enhanced chemical potential $\mu(L) \sim L$
2. Attractive potential, i.e., energy penalty to propagation

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- Analytics
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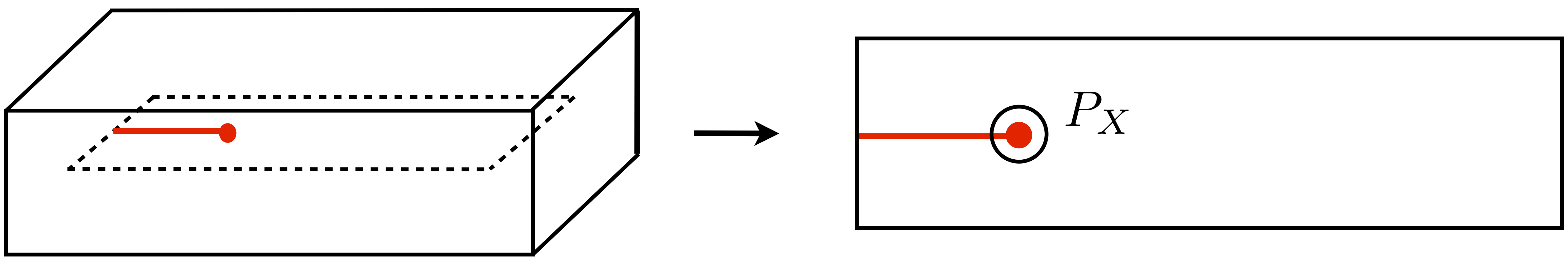
- Symm. protection?

Chemical potential

What are the logical operators of the coupled system?

➡ Logical operators T of the topological system since $[T,\, P_X] = 0$

$$H = -\sum_{X \subset \Lambda} P_X \otimes \mathbb{I}_{\mathcal{A}} + \sum_{X \subset \Lambda} g_X P_X \otimes \Theta_X + \mathbb{I}_{\mathcal{S}} \otimes H_{\mathcal{A}}$$



Auxiliary system is in thermal equilibrium $\rho_\Omega = e^{-\beta H_\Omega} / \mathrm{Tr}\left[e^{-\beta H_\Omega}\right]$

with effective Hamiltonian $\langle \Omega | H | \Omega \rangle$

Pedrocchi, Hutter, Wootton, Loss.
PRA **88**, 062313 (2013)

Chemical potential $\mu(L) \propto \mathrm{Tr}\left[\Theta_X \rho_\Omega\right]$ $\qquad \mu(L) \sim L$

If bath operator are unbounded, the chemical potential can diverge.

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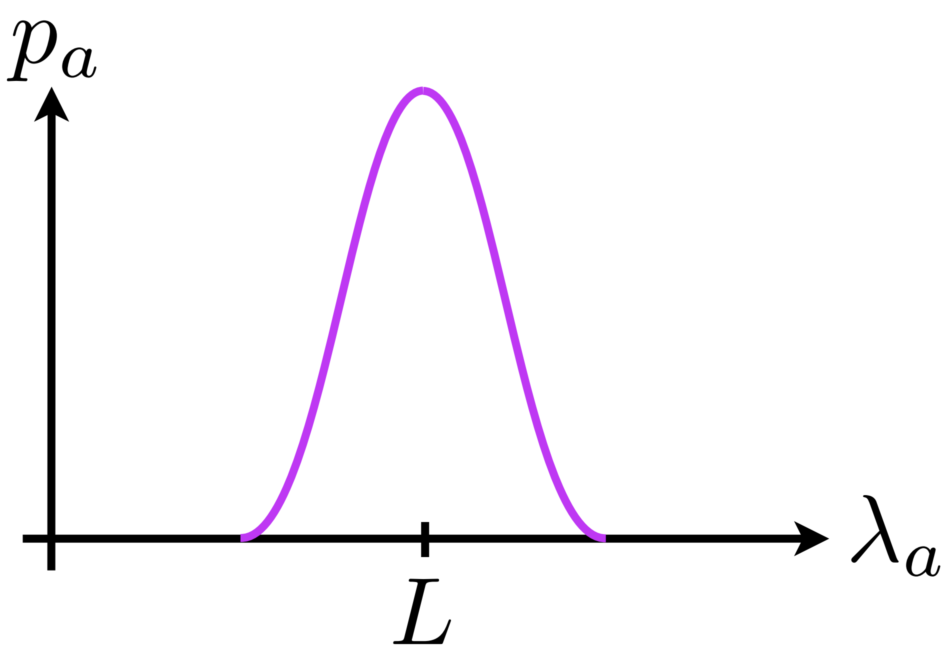
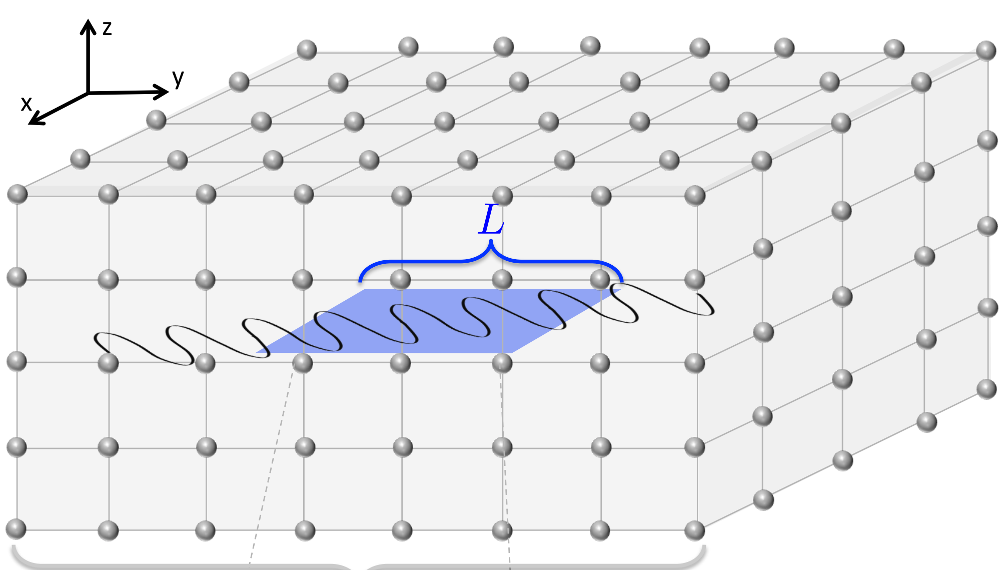
Diverging chemical potential

Chemical potential $\mu(L) \propto \mathrm{Tr}\left[\Theta_X \rho_\Omega\right]$

$$\Theta_X = \sum_a \lambda_a |a\rangle\langle a|$$

$$p_a \propto \langle a|e^{-\beta H_\Omega}|a\rangle$$





Correlation length of the auxiliary bath diverges.
➡ gapless auxiliary bath

Is the chemical potential scaling robust to perturbations?

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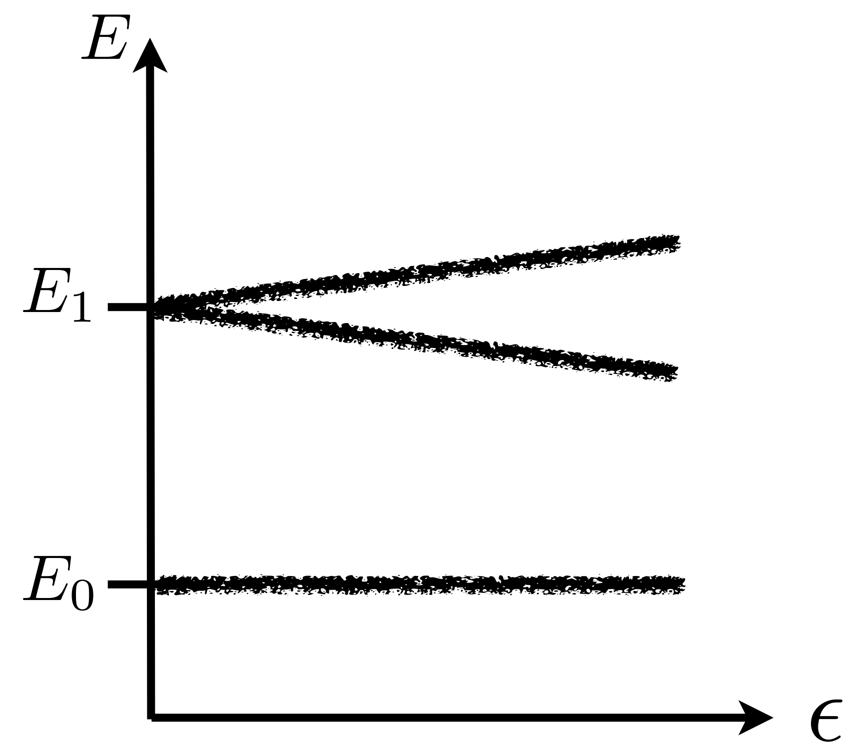
Chemical potential under perturbations?

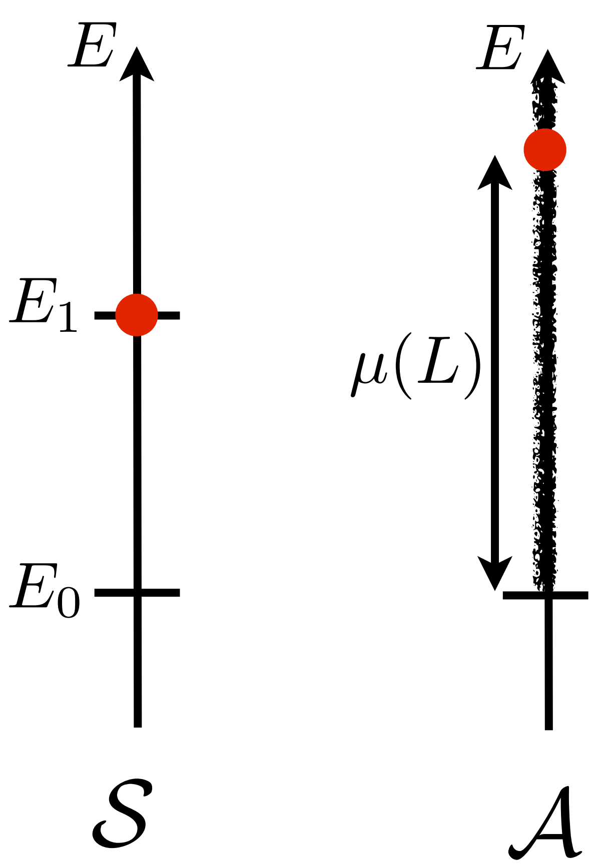
For topological system

$$|\mu(\epsilon) - \mu(0)| \leq \alpha\epsilon\mu(0)$$

$$\lim_{\epsilon\to 0} \frac{\mu(\epsilon)}{\mu(0)} = 1$$



Topological system coupled to a gapless auxiliary bath



$$\lim_{\epsilon\to 0} \lim_{L\to\infty} \frac{\mu(L,\epsilon)}{\mu(L,0)} \stackrel{?}{=} c$$

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Perturbations on the Hamiltonian of the auxiliary system

$$H \to H + \epsilon \sum_X P_X \otimes Q[\Theta_X]$$

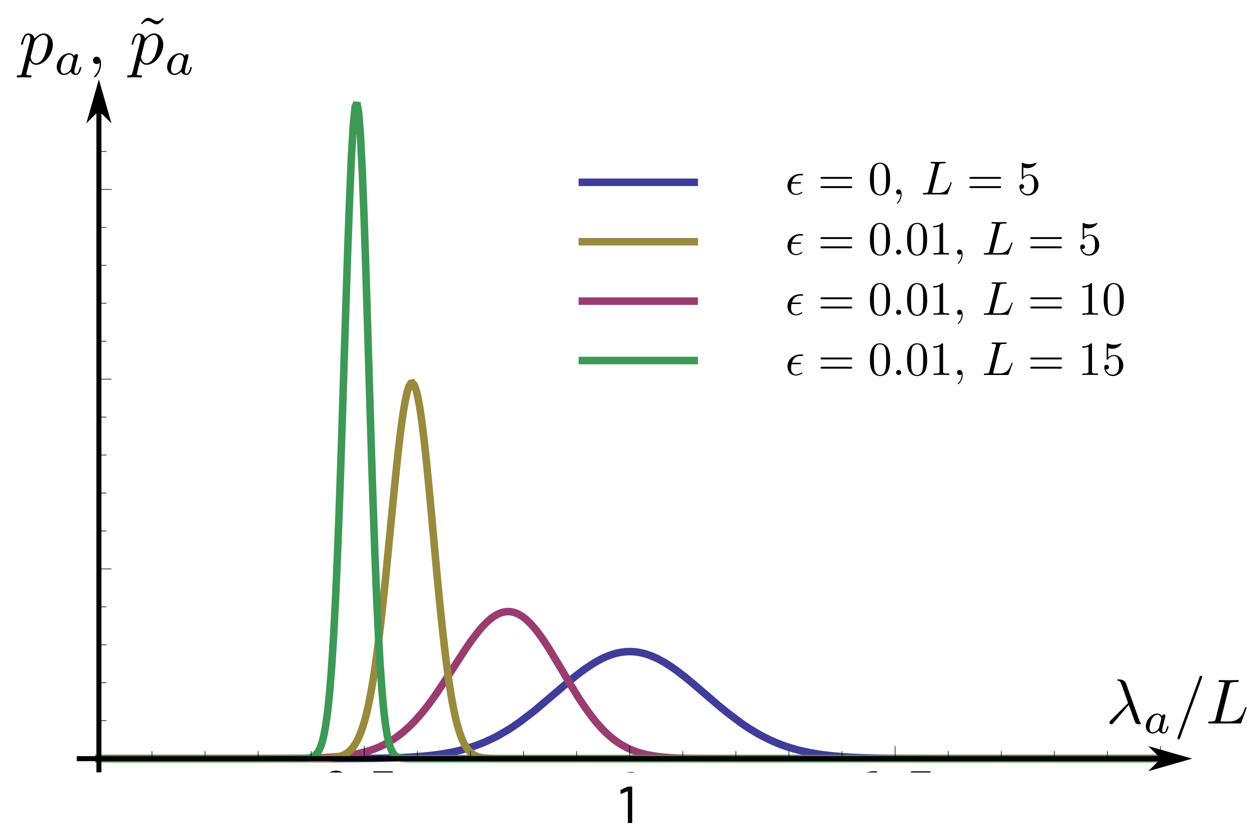
or

$$H_{\mathcal{A}} \to H_{\mathcal{A}} + \epsilon \sum_X Q[\Theta_X]$$

$$\Longrightarrow \quad H_\Omega \to \tilde{H}_\Omega = H_\Omega + \epsilon \sum_X Q[\Theta_X]$$

Q is a polynomial, e.g., $Q[\Theta_X] = \Theta_X^2$ or higher power. $\Theta_X = \sum_a \lambda_a |a\rangle\langle a|$

$$p_a \propto \langle a| e^{-\beta H_\Omega} |a\rangle \xrightarrow{\text{Golden-Thompson}} \tilde{p}_a \propto p_a e^{-\epsilon\beta Q[\lambda_a]}$$



In the limit of large system size, the chemical potential is bounded by a constant independent of system size.

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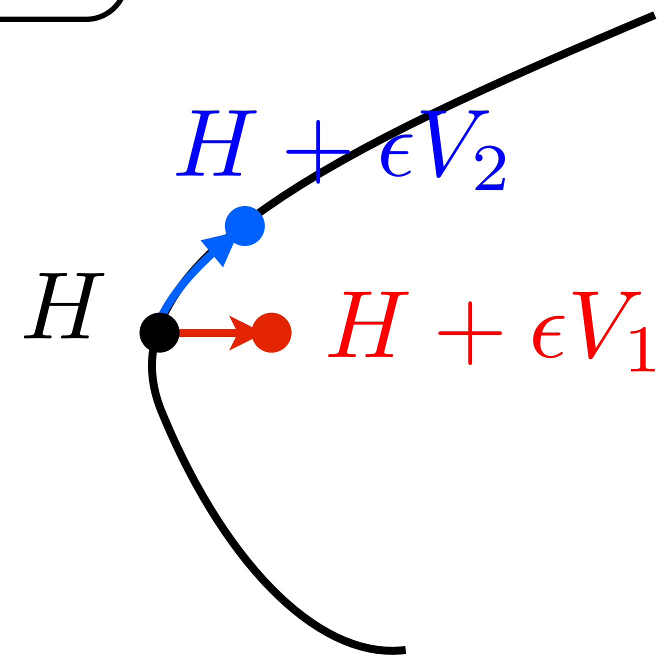
What we would like to prove

$$\boxed{\exists V \quad \forall \epsilon > 0 \quad \lim_{L \to \infty} \mu(L, \epsilon) \in \mathbb{R}}$$

???

Some perturbations will not change the scaling of the chemical potential.

Is there always a perturbation that reduces the chemical potential to a constant?



Could not prove it in the general case.

$$H = -\sum_{X \subset \Lambda} P_X \otimes \mathbb{I}_{\mathcal{A}} + \sum_{X \subset \Lambda} g_X P_X \otimes \Theta_X + \mathbb{I}_{\mathcal{S}} \otimes H_{\mathcal{A}}$$

Cases for which we can prove the existence of a suitable perturbation.

- Commuting case $[H, \Theta] = 0$ (+ Gaussian approximation)
- Massless scalar field + linear coupling

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Commuting case + Gaussian approximation

We are interested in the behavior of $\langle \Theta \rangle_\epsilon \equiv \dfrac{\mathrm{Tr}\left[e^{-\beta(H_\Omega + \epsilon V)}\Theta\right]}{\mathrm{Tr}\left[e^{-\beta(H_\Omega + \epsilon V)}\right]}$

$[H, \Theta] = 0 \longrightarrow \dfrac{d}{d\epsilon}\langle \Theta \rangle_\epsilon = -\langle \Theta Q[\Theta] \rangle_\epsilon + \langle \Theta \rangle_\epsilon \langle Q[\Theta] \rangle_\epsilon$

Gaussian approximation $\longrightarrow \langle (\Theta - \langle \Theta \rangle_\epsilon)^{2k+1} \rangle_\epsilon = 0$

Quadratic perturbation

$Q[\Theta] = \Theta^2$

$$\frac{d}{d\epsilon}\langle \Theta \rangle_\epsilon = -2\langle \Theta \rangle_\epsilon \left(\langle \Theta^2 \rangle_\epsilon - \langle \Theta \rangle_\epsilon^2 \right)$$

$$\frac{\langle \Theta \rangle_\epsilon}{\langle \Theta \rangle_0} = \exp\left(-2 \int_0^\epsilon \mathbb{V}(u) du \right)$$

Quartic perturbation

$Q[\Theta] = \Theta^4$

$$\frac{\langle \Theta \rangle_\epsilon}{\langle \Theta \rangle_0} \simeq \frac{1}{\sqrt{1 + 8\epsilon \mathbb{V}_0 \langle \Theta \rangle_0^2}} \xrightarrow[L \to \infty]{} 0$$

Extension to non-commuting case?

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Field theory perspective: massless scalar bosons

Lattice model

$$H = H_b + A \sum_p W_p \otimes (a_p + a_p^\dagger)$$

$$H_b = \epsilon_0 \sum_i a_i^\dagger a_i - t \sum_{\langle i,j \rangle} a_i^\dagger a_j$$

Fine-tuning condition $\epsilon_0 = 6t$

$$a_p + a_p^\dagger \longleftrightarrow \phi(x)$$

Field theoretic model

$$H = \int d^D x \left(\frac{1}{2} (\nabla \phi)^2 - w(x) \phi(x) \right)$$

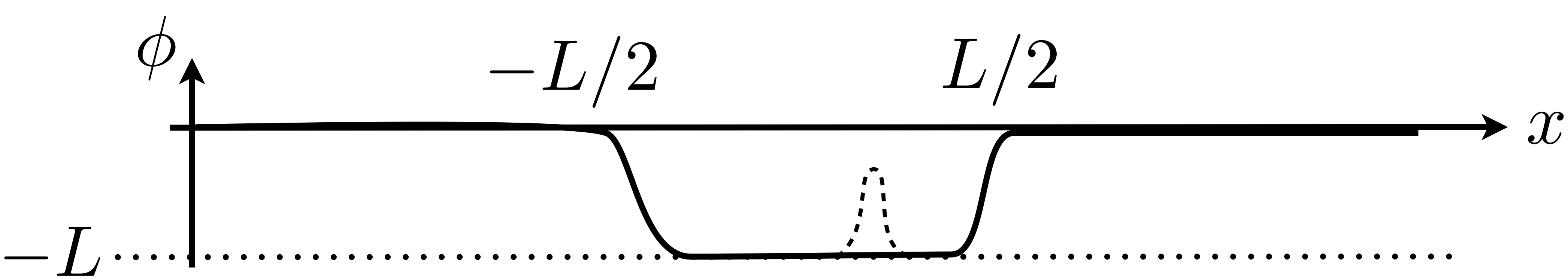
Equivalence $\theta_p \equiv a_p + a_p^\dagger$ $\quad \nabla \phi = \sum_{p' \in \mathcal{N}(p)} \theta_{p'} - \theta_p$

$$\frac{1}{2} (\nabla \phi)^2 = 6 \sum_p a_p^\dagger a_p - \sum_{p' \in \mathcal{N}(p)} a_p^\dagger a_{p'}$$ + higher order terms

QFT calculations

Field eq. = Poisson eq. with source term $\nabla^2 \phi = -w(x)$

Energy to add a quasiparticle $\mu \sim L$



Perturbation gives mass to particles

Lattice model

$$H_\varepsilon = \epsilon_0 \sum_i a_i^\dagger a_i - t \sum_{\langle i,j \rangle} a_i^\dagger a_j \boxed{+\varepsilon \sum_i a_i^\dagger a_i}$$

High occupation of bosonic modes is energetically penalized
➡Effective cutoff

Field theoretic model

$$H_\varepsilon = \int d^D x \left(\frac{1}{2} (\nabla \phi)^2 - w(x)\phi(x) + \varepsilon \phi^2 \right)$$

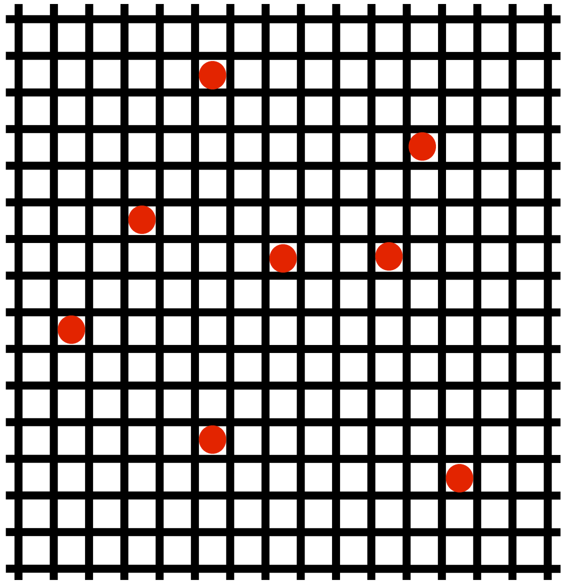
Bosons become massive.
➡Effective interaction becomes short-ranged

Can we find physical systems where
i) the masslessness of bosons is protected by symmetry?
ii) the chemical potential diverges with system size?

Systems with symmetry protection

- Gauge bosons, e.g. photons
- Goldstone bosons, e.g. phonons

$$J_{rr'} \propto \frac{1}{|r - r'|^\alpha}$$



α

$\alpha > D \qquad \mu(L) \in O(1) \qquad T_c = 0$

$D \qquad \mu(L) \sim \ln(L) \qquad \tau_{\mathrm{mem}} \sim poly(L) \qquad T_c \in O(1)$

$\alpha < D \qquad \mu(L) \sim L^{D-\alpha} \qquad T_c = \infty$

- Photons : coupling to charge

➡ charged anyons?

➡ Fractional quantum Hall effect

➡ Screening effects, stability?

- Phonons

➡ derivative coupling to anyons

➡ $\alpha = 3$

See discussion in Bonderson and Nayak PRB **87** 195451

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Conclusion

Introducing long-range interactions to stabilize quantum memories

General issue: can it be done in a way that is robust to perturbations?

In some cases, perturbations
- of the coupling between the topological system and the auxiliary bath
- of the Hamiltonian of the auxiliary bath

lead to change in the scaling of the chemical potential of the model.

Our work emphasizes the non-trivial interplay between robustness to perturbations and thermal stability in those proposals.

Thank you for your attention!

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