Quantum Error Correction for Long-Distance Quantum Communication

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Overview

- Old versus New Quantum Repeaters: QED vs. QEC
- Photon Loss Codes
- Ultrafast Long-Distance Quantum Communication
Overview

 ✓ Old versus New Quantum Repeaters: QED vs. QEC

 ✓ Photon Loss Codes

 ✓ Ultrafast Long-Distance Quantum Communication with Linear Optics
Classification of Quantum Repeaters

1.) Original Quantum Repeaters (Briegel et al., DLCZ,...):
   use entanglement distribution, swapping, purification
   (loss, local errors)

2.) Quantum repeaters with purification (loss) and QECC (local errors)

3.) Quantum repeaters with QECC only (loss and local errors)

Original Quantum Repeaters: Quantum Error Detection for Long-Distance Quantum Communication
Direct Transmission of Flying Qubits

\[ |\psi_{\text{in}}\rangle = \alpha |0\rangle + \beta |1\rangle \]

\[ \rho_{\text{out}} = \left( \alpha |0\rangle + \beta \sqrt{\eta} |1\rangle \right) \times \text{H.c.} + |\beta|^2 (1-\eta)|0\rangle\langle 0| \]

\[ \eta = \exp\left(-L/L_{\text{att}}\right) \]
Direct Transmission of Flying Qubits

\[ |\psi_{in}\rangle = \alpha |10\rangle + \beta |01\rangle \]

\[ \rho_{out} = \eta |\psi_{in}\rangle \langle \psi_{in}| + (1-\eta) |00\rangle \langle 00| \]

\[ \eta = \exp\left(-L/L_{att}\right) = F = \langle \psi_{in} | \rho_{out} | \psi_{in} \rangle \]
Direct Transmission of Flying Qubits

\[ |\psi_{\text{in}}\rangle = \alpha |10\rangle + \beta |01\rangle \]

\[ \rho_{\text{out}}^{\text{PS}} = \eta |\psi_{\text{in}}\rangle \langle \psi_{\text{in}}| \]

\[ \eta = \exp\left(-L/L_{\text{att}}\right) = P_{\text{succ}} = \text{Tr}(\rho_{\text{out}}^{\text{PS}}) \]
QED on Flying Qubits

\[ |\psi_{in}\rangle = \alpha |10\rangle + \beta |01\rangle \]
QED on Flying Qubits

\[ |\psi_{in}\rangle = \alpha |10\rangle + \beta |01\rangle \]

....need to detect the qubit non-destructively
Bell measurement detects syndrome and "recovers" in one step:
no loss = 2-photon detection,
photon lost = 1-photon detection
QED on Flying Qubits

Complications:

✓ on-demand generation of local Bell states
✓ Bell measurement with unit success probability
✓ never beats direct transmission
QED on Flying Qubits

\[ |\psi_{\text{in}}\rangle \xrightarrow{L_0} |\phi^+\rangle \xrightarrow{L_0} |\phi^+\rangle \xrightarrow{L_0} |\phi^+\rangle \xrightarrow{\text{classical channel}} \]

\[ P_{\text{suc}} = \left[ P_{\text{BM}} \exp\left( -\frac{L_0}{L_{\text{att}}} \right) \right]^{L/L_0} \leq \exp\left( -\frac{L}{L_{\text{att}}} \right) \]

(for any \( L_0 \))
Original Quantum Repeater

Essence of subexponential scaling:

some form of quantum error detection

and quantum memories
Original Quantum Repeater

✓ distribute known, entangled states
✓ distribute different copies in each segment
✓ QED/entanglement purification
✓ quantum memories
✓ two-way classical communication
With Memories: Quantum Repeater

\[
\begin{align*}
\text{Rate} & \sim P_{\text{distr}} \left( \frac{2}{3} P_{\text{swap}} \right)^{\log_2 (L/L_0)} \\
& \sim \left( \frac{L}{L_0} \right)^{\log_2 (2/3 P_{\text{swap}})}
\end{align*}
\]
Without Memories: Quantum Relay

\[ P_{\text{distr}} \]

\[ L_0 \]

\[ P_{\text{swap}} \]

\[ L \]

\[ \text{Rate} \sim P_{\text{distr}}^{L/L_0} \cdot P_{\text{swap}}^{L/L_0 - 1} \]
Original Quantum Repeater

- Entanglement Distribution
- Entanglement Purification (Quantum Error Detection)
- Entanglement Swapping
- Quantum Memories

H.J. Briegel et al., PRL 81, 5932 (1998)
DLCZ Quantum Repeater


$I \otimes \eta \rightarrow |00\rangle + r |11\rangle$

$\eta \otimes I \rightarrow |00\rangle + r |11\rangle$

$|01\rangle \pm |10\rangle$

$P_{\text{distr}} \sim \eta \cdot r^2$; \quad $F \approx 1 - O(r^2)$
DLCZ Quantum Repeater

\[ \approx |0000\rangle + r \sqrt{1 - \eta} (|1\rangle + |0\rangle) + O(r^2) \]

\[ r \sqrt{1 - \eta} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]

\[ I \otimes \eta \rightarrow |00\rangle + r |1\rangle \]

\[ \eta \otimes I \rightarrow |00\rangle + r |1\rangle \]

\[ |01\rangle \pm |10\rangle \]

\[ P_{\text{distr}} \sim \eta \cdot r^2 ; \quad F \approx 1 - O(r^2) \]
loss space: only QED, not QEC!

$$\cong |0000\rangle + r\sqrt{\eta} (|1100\rangle + |0011\rangle) + O(r^2)$$

$$r\sqrt{1-\eta} \rightarrow \begin{array}{c} 1000 \\ 0001 \end{array}$$

$$I \otimes \eta \rightarrow |00\rangle + r|11\rangle$$

$$\eta \otimes I \rightarrow |00\rangle + r|11\rangle$$

$$|01\rangle \pm |10\rangle$$

$$P_{\text{distr}} \sim \eta \cdot r^2; \quad F \approx 1 - O(r^2)$$
Original Quantum Repeater

- distribute known, entangled states
- distribute different copies in each segment
- QED/entanglement purification
- quantum memories
- two-way classical communication

Problems: very slow, limited by CC rates, good memories required

Rate $\leq c / L$  
\[ \text{e.g. } 100\text{Hz}/1000\text{km} \sim \frac{1}{O(\text{poly}(L))} \]
New Quantum Repeaters: Quantum Error Correction for Long-Distance Quantum Communication
Encoded Quantum Repeaters: Local Errors

1. Encoded Generation

2. Encoded Connection

3. Pauli Frame

\[
\begin{align*}
\bar{0} &= |000\rangle, \quad \bar{1} = |111\rangle \\
\text{etc.}
\end{align*}
\]


S. Bratzik, H. Kampermann, and D. Bruß, PRA 89, 032335 (2014) implementation-independent

N.K. Bernardes and P.v.L., PRA 86, 052301 (2012) secret key rates in QKD

HQR with encoding
Encoded Quantum Repeaters: Loss Errors

W.J. Munro et al., Nature Photon. 6, 777 (2012)

topological surface codes
parity loss codes
cluster states and feedforward
Photon Loss Codes

Leung’s bosonic code:
\[
|\bar{0}\rangle = \frac{|00\rangle + |04\rangle}{\sqrt{2}}, \quad |\bar{1}\rangle = |22\rangle
\]

exact

Leung’s [4,1] AD code:
\[
|\bar{0}\rangle = \frac{|0000\rangle + |1111\rangle}{\sqrt{2}},
\]
approximate
\[
|\bar{1}\rangle = \frac{|0011\rangle + |1100\rangle}{\sqrt{2}}
\]
Photon Loss Codes

Quantum Parity Code (QPC):

\[ |\pm(n,m)\rangle = \left( |0\rangle^m \pm |1\rangle^m \right)^\otimes n \]

\[ |0\rangle^m = |10\rangle^\otimes m, \quad |1\rangle^m = |01\rangle^\otimes m \]

\[ |\bar{0}\rangle^{(n,m)} = \left( |+\rangle^{(n,m)} + |\rangle^{(n,m)} \right) \quad \text{and} \quad |\bar{1}\rangle^{(n,m)} = \left( |+\rangle^{(n,m)} - |\rangle^{(n,m)} \right) \]

QPC\((n,n)\) corrects \((n - 1)\) photon losses

Photon Loss Codes

QPC(1,1):

\[ |\pm\rangle^{(1,1)} = \frac{(|10\rangle \pm |01\rangle)}{\sqrt{2}} \], \quad |\bar{0}\rangle^{(1,1)} = |10\rangle, \quad |\bar{1}\rangle^{(1,1)} = |01\rangle

QPC(2,2):

\[ |\bar{0}\rangle^{(2,2)} = \frac{|10101010\rangle + |01010101\rangle}{\sqrt{2}}, \quad |\bar{1}\rangle^{(2,2)} = \frac{|10100101\rangle + |01011010\rangle}{\sqrt{2}} \]

\[ C_{QPC(2,2)} = C_{[4,1]} \circ C_{\text{Dual–Rail}} \quad \text{(is exact!)} \]
Photon Loss Codes

Quantum Parity Code (QPC):

\[ Z_{ij} Z_{i,j+1} \quad i = 1 \ldots n, \quad j = 1 \ldots (m - 1) \]
\[
\prod_{j=1}^{m} X_{ij} X_{i+1,j} \quad i = 1 \ldots (n - 1) 
\]
\[ n(m - 1) + n - 1 = nm - 1 \quad \text{independent stabilizers} \]

QPC(2,2):
\[
\langle Z_{12} Z_{21}, Z_{22} Z_{21}, X_{11} X_{21} X_{12} X_{22} \rangle = \langle ZZII, IIZZ, XXXX \rangle 
\]
like [4,1] code

Ultrafast Quantum Communication

...replace DR-qubit/Bell states/BM’s by QPC-encoded qubit/Bell states/BM’s, use stabilizer formalism and exploit transversality of QPC code as a CSS code.

Ultrafast Quantum Communication

\[ \psi_{\text{in}}^{(n,m)} \]

\[ \phi^+ \]

\[ \text{classical channel} \]

\[ L_0 \]

BM

BM

BM

\[ (n,m) \]

\[ (n,m) \]

\[ (n,m) \]

...many physical BM’s for one logical BM via many physical CNOTs and many physical Hadamards: need nonlinear operations, matter-light interactions,...
Ultrafast Long-Distance Quantum Communication with Linear Optics
...replace matter-qubit-based QPC-Bell states by optical QPC-Bell states and nonlinear light-matter interactions by static linear optics.
...replace matter-qubit-based QPC-Bell states by optical QPC-Bell states and nonlinear light-matter interactions by static linear optics

\[
\psi_{\text{in}}^{(n,m)} L_0 |\phi^+\rangle^{(n,m)} \xrightarrow{BM} |\phi^+\rangle^{(n,m)} \xrightarrow{BM} |\phi^+\rangle^{(n,m)} \xrightarrow{BM}
\]

classical channel

\[
P_{\text{succ}} = \left[ \sum_{l=0}^{nm} P_{BM,l} \binom{nm}{l} \eta^{nm-l} (1-\eta)^l \right]^{L/L_0} \quad \eta = \exp\left(-\frac{L_0}{L_{\text{att}}}\right)
\]
Linear-Optics Quantum Communication

...replace matter-qubit-based QPC-Bell states by optical QPC-Bell states and nonlinear light-matter interactions by static linear optics

What is \( P_{BM,l} \)?

Can we again exploit „transversality“?

...
Linear-Optics Quantum Communication

BM of QPC(2,2) encoded Bell states:
BM of QPC(2,2) encoded Bell states:
Linear-Optics Quantum Communication

QPC-encoded BM works asymptotically well with linear optics (no loss) and it even still works in the presence of losses!

\[ P_{BM,l=0} = 1 - 2^{-n} \]

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<th>2</th>
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Table 1: Success probabilities \(p_l\) in % of the one-sided (top) and the symmetric BM (bottom) given the total number of photons lost \(l\).
Linear-Optics Quantum Communication
Success Probabilities (Temporal Cost)

Rate $R = \frac{P_{\text{succ}}}{t_0}$

1000 km
Success Probabilities (Temporal Cost)

10000 km

QPC(21,6)  

QPC(31,7)
Success Probabilities (Temporal Cost)

150000 km
Total Cost

\[ C = \left( \frac{2nm}{R} \right) \frac{L}{L_0} = \left( \frac{2nmt_0}{P_{\text{succ}}} \right) \frac{L}{L_0} \]

Total Cost

$\text{QPC}(21,6)$

$\text{QPC}(31,7)$

1000 km
Summary

☑ Standard quantum repeaters using QED are scalable in principle, but slow

☑ New generation of quantum repeaters using QEC significantly improve the rates

☑ Ultrafast loss-code-based scheme is implementable with linear optics

QR rates near CC rates, only limited by local times