Error Correction in a Fibonacci Anyon Code

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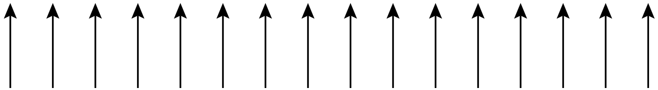
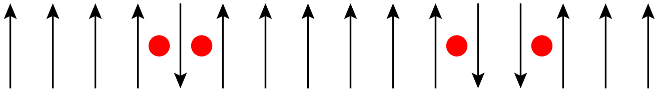
Error Correcting Code

Store one bit in a global state:

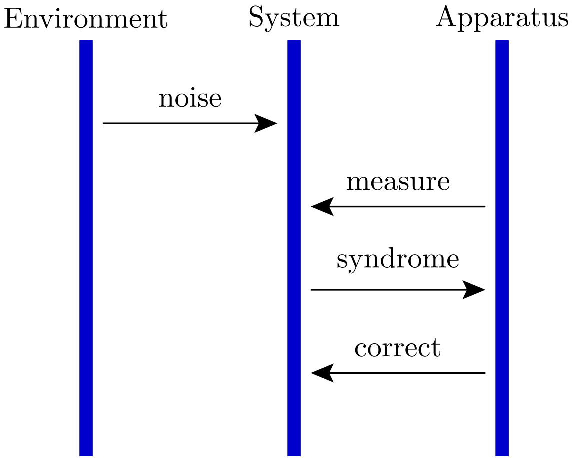


Noise acts locally:



Error correction: join nearby frustrations

Simulation


Environment
System
Apparatus
noise
measure
syndrome
correct

History

Dennis, Kitaev, Landahl, Preskill (2002) *PMA*

Duclos-Cianci, Poulin (2010) *Soft-RG*

Bravyi, Haah (2013) *Hard-RG*

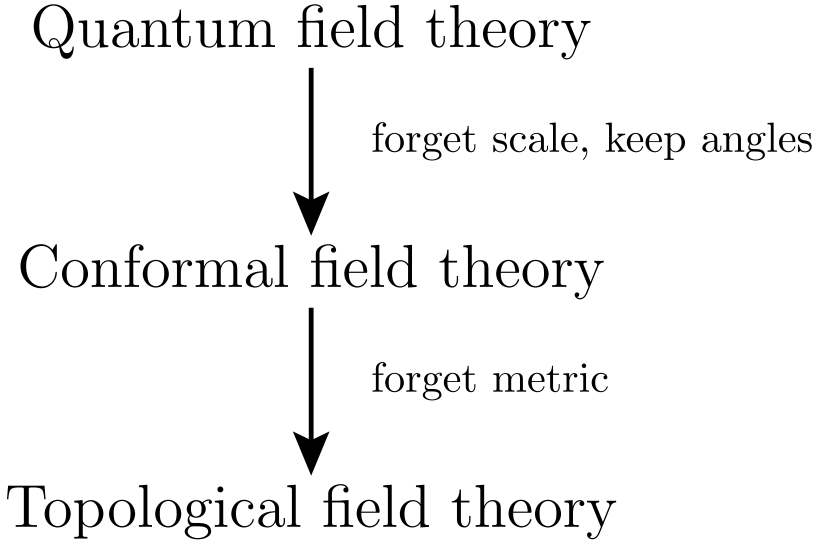
Anwar, Brown, Campbell, Browne (2014) *qudits*

Wootton, Burri, Iblisdir, Loss (2014) *non-Abelian*

Brell, Burton, Dauphinais, Flammia, Poulin (2014) *non-Abelian*

Mathematics:

the art of forgetting


Quantum field theory
forget scale, keep angles
Conformal field theory
forget metric
Topological field theory

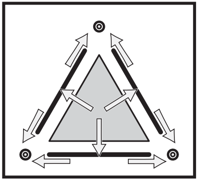
Elementary Applied Topology

ROBERT GHRIST

From “Elementary Applied Topology”, by Robert Ghrist:

The collection of chains and boundary maps is assembled into a **chain complex**:

$$\cdots \longrightarrow C_k \xrightarrow{\partial_k} C_{k-1} \xrightarrow{\partial_{k-1}} \cdots \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0 \xrightarrow{\partial_0} 0\ . \qquad (4.1)$$

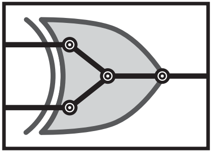


The chain complex is **graded**, in this case by the dimension of the cells. It is beneficial to denote the chain complex as a single object $\mathcal{C} = (C_\bullet, \partial)$ and to write ∂ for the boundary operator acting on any chain of unspecified grading. Chain complexes are a representation of a cell complex within linear algebra. It seems at first foolish to algebraicize the problem in this manner – why bother with vector spaces which simply record whether a cell is present (1) or absent (0)? Why express the boundary of a cell in terms of linear transformations, when the geometric meaning of a boundary is clear? By the end of this chapter, probably, and the next, certainly, this objection will have been forgotten.

From "Elementary Applied Topology", by Robert Ghrist:

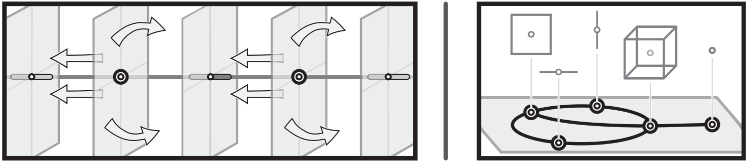
Example 9.4 (Logic gates) ⊙

Consider a simple XOR gate, with two binary inputs and one binary output given by exclusive conjunction. Topologize this gate as a directed Y-graph Y. Let $\mathcal{F}$ be the sheaf taking values in $\mathbb{F}_2$ vector spaces over Y with stalk dimension one everywhere except at the central vertex, where it equals two.



The restriction maps from the central vertex to the three edges are as follows. On the two input edges, the restriction map is projection to the first and second factors, respectively. The restriction map to the output edge is addition: $+\colon \mathbb{F}_2^2 \to \mathbb{F}_2$. This instantiates an exclusive-OR gate – the global sections correspond precisely to the truth table of inputs and outputs. A similar approach does *not* work for an AND gate, since the operation $\mathbb{F}_2^2 \to \mathbb{F}_2$ encoded by AND is no longer a homomorphism; neither is the involutive NOT, nor OR, nor NOR, nor NAND. See [159, 254] for other approaches to sheaf circuitry. ⊙

From “Elementary Applied Topology”, by Robert Ghrist:



The stalk of this sheaf encodes local H^n. For example, on an n-dimensional manifold, this process yields a constant sheaf (of dimension one), called the **orientation sheaf**, *cf.* Example 4.18. The manifold is orientable if and only if the orientation sheaf has a global section. For a finite graph, the local H^1 sheaf has stalk dimension equal to 1 on edges and equal to $\deg(v) - 1$ on each vertex v. The restriction maps

“Elementary Applied Topology”, by Robert Ghrist:

Manifolds
Complexes
Euler Characteristic
Homology
Sequences
Cohomology
Morse Theory
Homotopy
Sheaves
Categorification

"Topological features are robust. The number of components or holes is not something that should change with a small error in measurement. This is vital to applications in scientific disciplines, where data is never not noisey."

– Robert Ghrist

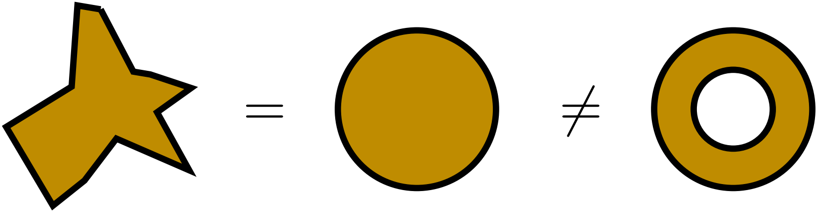
Cartoon

Topological

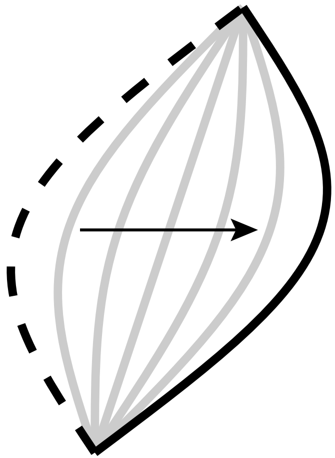
Quantum

Field Theory

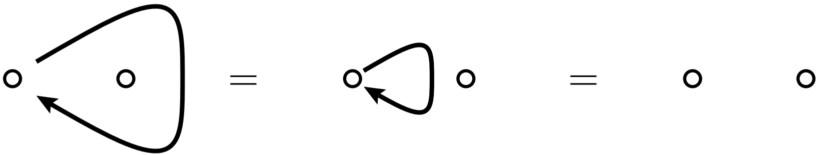
Topological



Isotopy

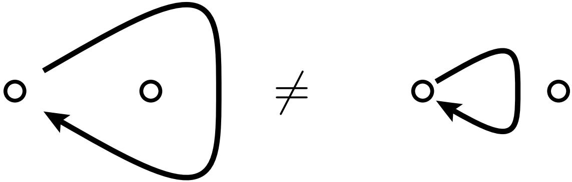


Particle exchange in 3d

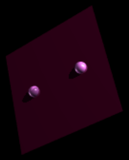


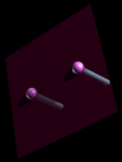
Swap: $S|\psi\rangle = |\psi\rangle$ ...Bosons
Or, swap: $S|\psi\rangle = -|\psi\rangle$ ...Fermions

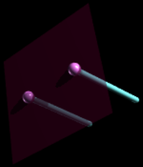
Particle exchange in 2d

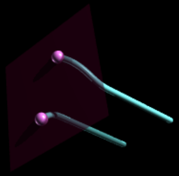


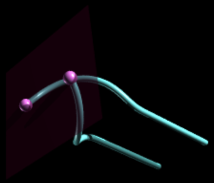
World lines: (2+1)d

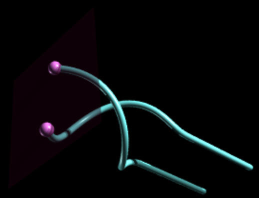


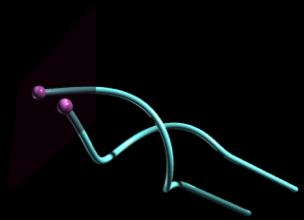


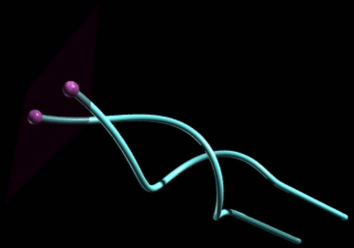


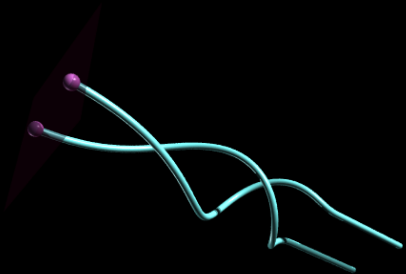


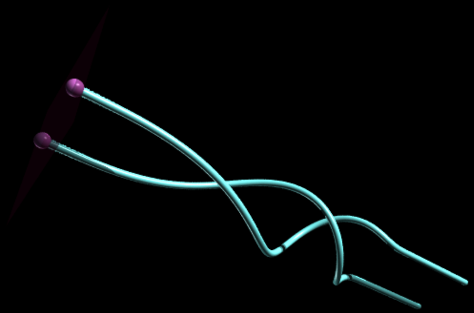








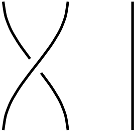


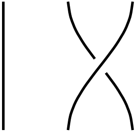


Braid group

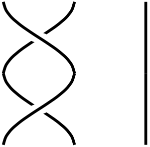
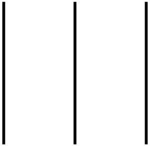
$\hookrightarrow$ braid group on three strands, B_3

$\hookrightarrow$ has two generators

$\sigma_1 =$ 

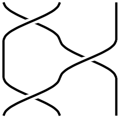
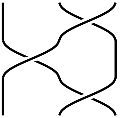
$\sigma_2 =$ 

$\hookrightarrow$ has inverses

$\sigma_1^{-1}\sigma_1 =$  $=$ 

Braid group

$\hookrightarrow$ B_3 has one non-trivial relation

$\sigma_1\sigma_2\sigma_1 =$  $=$  $= \sigma_2\sigma_1\sigma_2$

Why do we care?

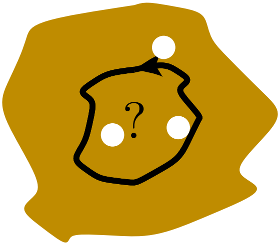
Braid group acts on states, for example:

$$|\psi\rangle \rightarrow \sigma_1|\psi\rangle$$

But what are the states?

Observables

$\hookrightarrow$ measure charge in a region

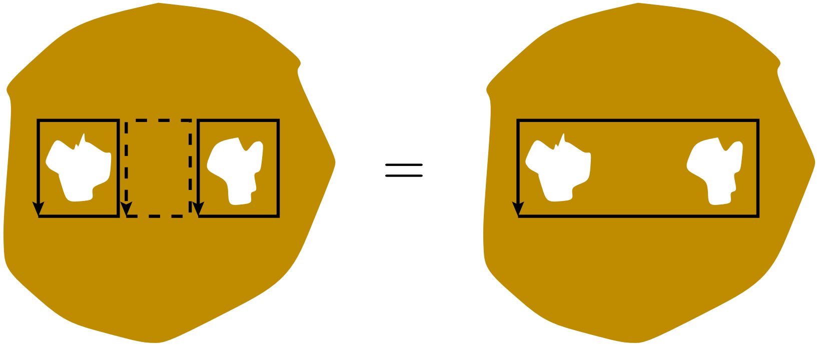


$\hookrightarrow$ Example charge values: $\mathbb{A} = \{I, \tau\}$.

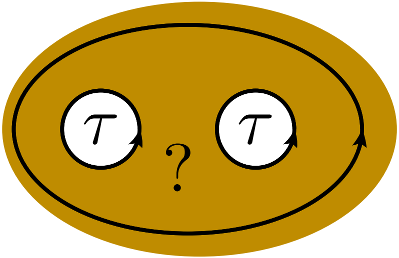
$\hookrightarrow$ superposition of these:

$$|\psi\rangle = \alpha|I\rangle + \beta|\tau\rangle.$$

Abelian (Homology)



Non-Abelian: Fibonacci anyons



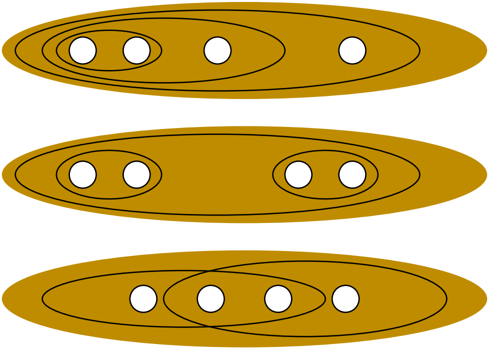
$\hookrightarrow$ could be either τ or I

$\hookrightarrow$ state space of two τ charges is 2-dimensional

$\hookrightarrow$ called the "fusion" space

Commutation

$\hookrightarrow$ non-overlapping observables commute

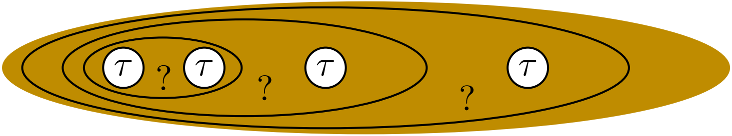


$\hookrightarrow$ a maximal collection of commuting observables corresponds to a pair-of-pants decomposition of the space

Fibonacci anyons

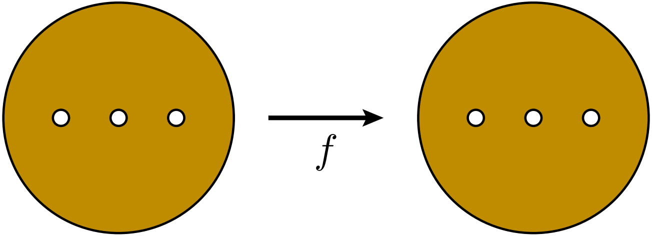
$\hookrightarrow$ dimensionality grows like the Fibonacci Sequence:

$$1, 1, 2, 3, 5, ...$$



Topological symmetry

Look at self-maps (homeomorphisms) of this surface:



Topological symmetry

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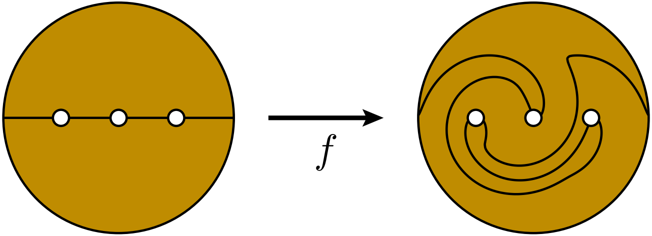
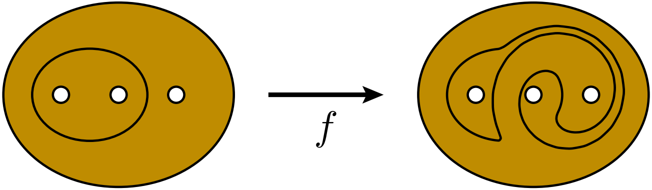


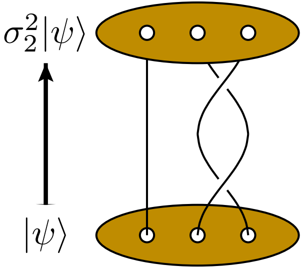
These maps form a group called the *mapping class group*.

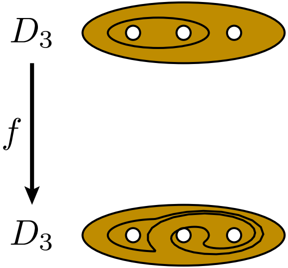
Why do we care?

The mapping class group acts on observables:



Choose your picture

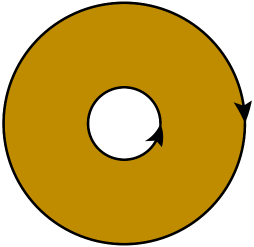




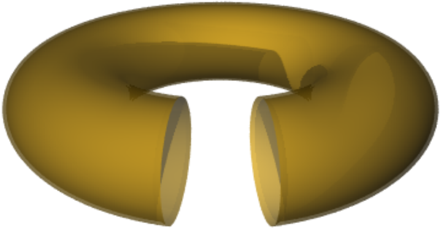
Braid group acts on states: "Schrodinger picture"

Mapping class group acts on observables: "Heisenberg picture"

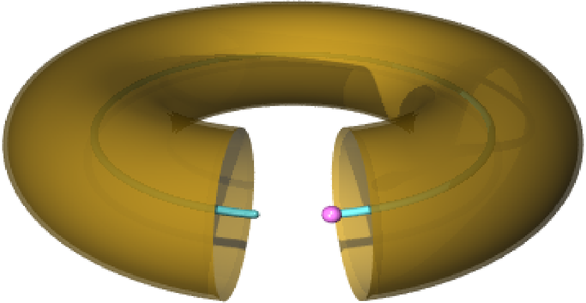
The code



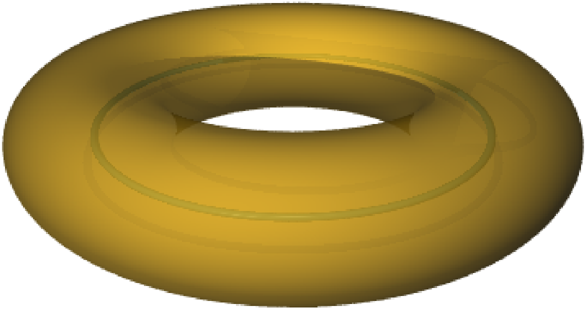
$=$



The code



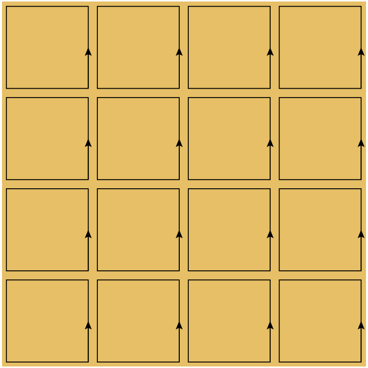
The code



$$|\psi\rangle = \alpha|I\rangle + \beta|\tau\rangle$$

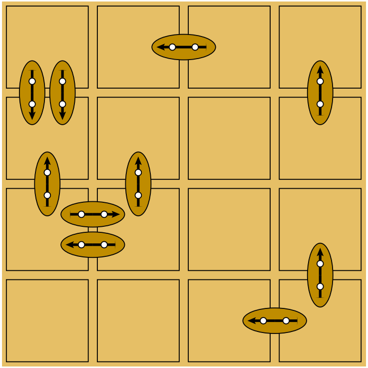
Syndrome measurement

$\hookrightarrow$ Measure charge inside tiles



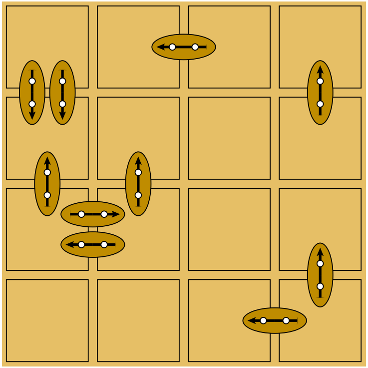
Error process

$\hookrightarrow$ poisson process pair creation



Error process

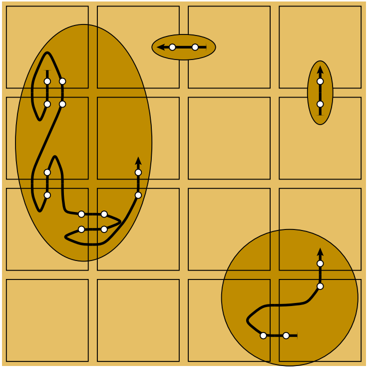
$\hookrightarrow$ problem scales exponentially, so decompose into pieces



$\hookrightarrow$ a bunch of disjoint copies of a disc with 2 holes

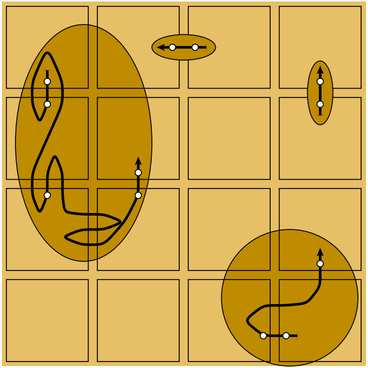
Join

$\hookrightarrow$ merge (sew) discs that participate in the same tile



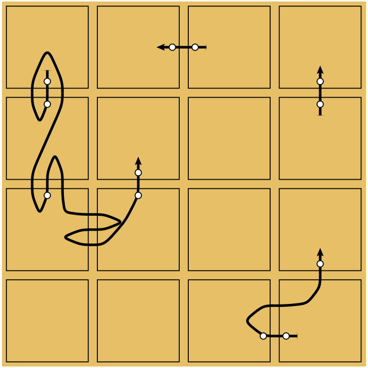
Syndrome

$\hookrightarrow$ measure the charge on each square



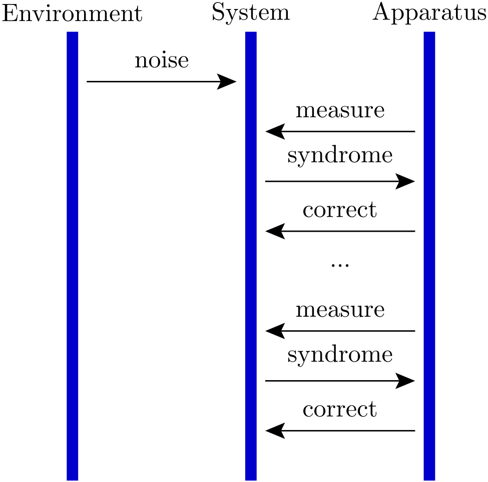
Error correction

↪ cluster nearby charges

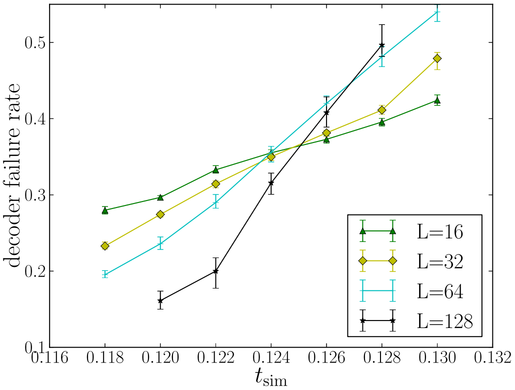


↪ succeed if we don't wrap around the torus

Simulation


Environment
System
Apparatus
noise
measure
syndrome
correct
...
measure
syndrome
correct

Monte-Carlo



$\hookrightarrow$ threshold at $t_{sim} \simeq 0.125$

Bad news:

Problem scales exponentially.
On 128x128 lattice with error rate 12.5%, dimension $\sim 1.6^{2048}$.
Braiding operations are dense on this space.

Good news:

Below percolation threshold, clusters of size $O(log(n))$.

Bad news:

Decoding joins clusters.

Good news:

Decoding fuses charges.

Summary

$\hookrightarrow$ Can do error correction in an arbitrary TQFT

$\hookrightarrow$ Simulation of system is possible by decomposing

$\hookrightarrow$ Combinatorial formulation of problem

Question:

$\hookrightarrow$ What is the scaling of this numerical simulation?